

DESIGN OF PR FRAMES WITH TOP AND SEAT ANGLE CONNECTIONS USING THE DIRECT ANALYSIS METHOD

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ABSTRACT: Partially restrained (PR) connections are connections that possess stiffness and moment characteristics that fall between the extreme cases of pinned and rigid. This paper proposes a design methodology for PR frames that takes into consideration the semi-rigid nature of the connections, including their loading/unloading behavior under combined gravity and wind loads. Top and seat angle connections, modeled using the three-parameter power model, are used to demonstrate the proposed design procedure. To simplify the design, two linearized connection stiffness values calculated on the basis of expected connection loads are used. The analysis is carried out using the American Institute of Steel Construction (AISC) direct analysis method in which notional horizontal loads, expressed as a fraction of the gravity load, are applied to the PR frames in conjunction with the use of reduced member axial and bending stiffness. Examples are given to demonstrate the validity of the proposed method of PR frame design.

Keywords: Partially restrained frames, Top and seat angle connections, Direct analysis method, Steel structures, Design and analysis

1. INTRODUCTION

Connections are important structural elements in a frame structure. The behavior of the frame is very much affected by the types of connections used. If the connection possesses sufficient strength and stiffness, full continuity between the connected elements can be assumed for analysis and design. However, if either the connection strength or stiffness falls short of what is assumed, allowances must be made in the analysis and design processes to ensure that the frame will behave as expected. In a conventional analysis and design of steel frames, a simplifying assumption that the beam-to-column connections are either fully rigid or ideally pinned is often used. The rigid joint assumption implies that full slope continuity exists between the adjoining members, which means the angle between the beam and column remains more or less unchanged as the frame deforms; and the full or a substantial amount of moments is transferred between the beam and the column. On the other extreme, the assumption of an ideally pinned connection implies that no or very little moment will be transmitted between the beam and the column and as far as rotation is concerned the beam and column will behave independently.

Although the use of these idealized joint behavior drastically simplifies the analysis and design procedures, the validity of these assumptions becomes questionable for cases in which the rigidity of connections, which can be considered as the slope of the relation between the connection moment M and the connection relative rotation θ , is intermediate between the fully rigid and ideally pinned cases. These connections, referred to as semi-rigid or partially restrained (PR) connections, have been the subject of research for many years, and various researchers have proposed analysis and design methods for frames that utilize these PR connections (Ang and Morris [1], Lui and Chen [2,3,4], Chen [5], Cunningham [6], Wu and Chen [7], Barakat and Chen [8,9], Hsieh and Deierlein [10], Lui [11], Xu [12], Surovek et al. [13]). Detailed summaries of research on PR connections and frames can be found in a number of books and monographs such as those by Narayanan [14], Lorenz et al. [15], Chen et al. [16], Bjorhovde et al. [17], Chan and Chui [18], Chen [19], Faella et al. [20], Chen et al. [21], etc., and will not be repeated here.

According to the American Institute of Steel Construction (AISC [22]) specification, a connection is considered to be a PR connection if its secant stiffness under service load R_{ks} falls in the range $2EI/L < R_{ks} < 20EI/L$, where EI and L are the flexural rigidity and the length of the adjoining beam, respectively. Despite acknowledging the existence of PR connections in frame structures and suggesting that the relevant response characteristics of PR connections must be included in the analysis of the structure for member and connection forces, displacements and frame stability, no recommendations are currently given in the AISC specification on how these PR frames should be designed. In addition, for the first time AISC is espousing the direct analysis method as the main design method for stability, while the commonly-used effective length method, first-order analysis method, and approximate second-order analysis using moment magnification (B_1 , B_2) factors have been relegated to the appendix. It is therefore the objective of this paper to propose a design method for PR frames using the direct analysis approach.

2. CONNECTION TYPES, CLASSIFICATIONS AND BEHAVIOR

There are various means by which connections can be classified. For instance, they can be classified by the connecting medium used, e.g., bolted versus welded connections. They can also be classified by their functionality, e.g., beam-to-column connections, hanger connections, bracing connections, truss connections, etc.; or by the type of internal forces/moment that will be transmitted, e.g., shear versus moment connections; or by the types and geometries of connection elements used, e.g., single plate connections, double web angle connections, top and seat angle connections, flange-angle connections, end plate connections, flange-plated connections, etc. Regardless of how connections are classified, the behavior of connections is often described by their moment-rotation (M - θ) response as shown in Figure 1. Although connections do experience axial, shear and torsional deformations, they are usually small compared to the rotational deformation. Consequently, researchers often consider only the connection's rotational deformation in assessing frame response. Aside from a few exceptions (Cerfontaine and Jaspart [23], Urbonas and Daniunas [24,25]), almost all experiments on connections conducted in the past few decades have focused primarily on their flexural response, and over the years several connection database and various connection models have been developed for general use (Frye and Morris [26], Goverdhan [27], Ang and Morris [1], Nethercot [28], Kishi and Chen [29], Chen and Kishi [30], Chen et al. [21]). From these database, it can be observed that (1) almost all types of connections exhibit a M - θ behavior that falls between the extreme cases of ideally pinned and fully rigid conditions, (2) except for small values of moment the M - θ relationships are typically nonlinear over a large range of loading, and (3) the maximum moment a connection can transmit often decreases with the more flexible connection (i.e., there is a correlation between strength and stiffness).

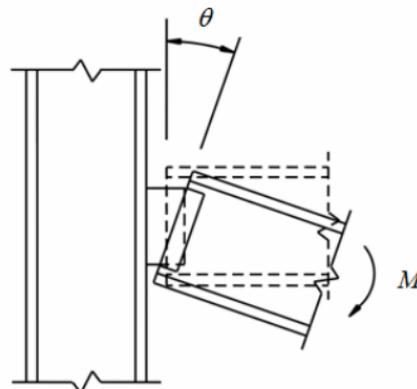


Figure 1. Flexural (or Rotational) Deformation of a PR Connection under a Moment

Because of the above observations, connections can also be classified according to their stiffness or strength (see for example Bjorhovde et al. [31], Eurocode 3 [32], Nethercot et al. [33], AISC [22]). Since the AISC direct analysis procedure will be used, the AISC connection classification scheme will be adopted for the present study. For a connection to be classified as a PR connection, its secant stiffness R_{ks} under service load (i.e., the slope of a straight line drawn from the origin to the moment M_s under service load condition as shown in Figure 2) falls in the range $2EI/L < R_{ks} < 20EI/L$, where EI and L are the flexural rigidity and the length of the beam, respectively. In terms of strength, a PR connection should be able to transmit at least 20% of the full plastic moment of the beam when it is experiencing a rotation $\theta = 0.02$ radian. Furthermore, it should possess sufficient rotational capacity so that θ_u , defined as the connection rotation where its strength has dropped to 80% of its peak value as shown in Figure 2 or a rotational deformation equal to 0.03 radian if the connection experiences little or no loss in strength, will exceed what is required for design at the strength limit state.

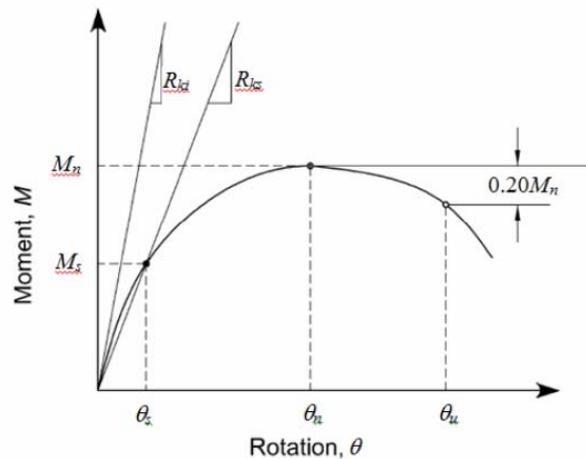


Figure 2. Moment-rotation Response of a PR Connection

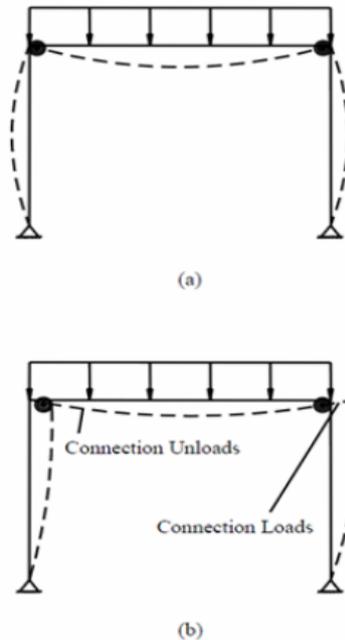


Figure 3. Loading/Unloading Behavior of PR Connections –
 (a) Both Connections Load under Gravity Load,
 (b) Windward Connection Unloads while Leeward Connection Loads under Wind Load

Another aspect of PR connections that needs to be addressed is their loading/unloading response to load sequencing. With reference to Figure 3 in which a simple portal frame is shown with PR connections present at both beam-column joints. When the gravity load is applied, both connections undergo loading. However, when a lateral load such as wind is applied, the windward connection will experience unloading while the leeward connection will continue to undergo loading. Depending on the difference between the loading and unloading stiffness, the forces and moments developed in the frame can be quite different. The design procedure for PR frame to be discussed in a later section will account for this connection loading/unloading response.

3. CONNECTION MODEL

Almost all $M-\theta$ data for PR connections are obtained from laboratory tests. These test data are then curve-fitted to provide empirical equations for use in analysis and design. Many such equations in the form of linear, bilinear, trilinear, polynomial, spline, exponential, and power functions have been proposed (Frye and Morris [26], Goverdhan [27], Ang and Morris [1], Nethercot [28], Kishi and Chen [29], Chen and Lui [34]), and computer programs (PRCONN [35], Chen and Toma [36], Chen et al. [21]) are available from which information on a number of PR connection types can be retrieved. In the present study, the three-parameter power model (Kishi et al. [37,38], Kishi and Chen [39]) for connections made from angle sections will be used. In this model the initial connection stiffness R_{ki} and ultimate moment capacity M_{ult} of the connections were determined using mechanistic models. Using these two quantities plus a curve-fitting shape factor n , the experimental data were represented by an equation in the form (Richard and Abbott [40])

$$M = \frac{R_{ki}\theta}{\left[1 + \left(\frac{\theta}{\theta_o}\right)^n\right]^{1/n}} \quad (1)$$

where

R_{ki} = initial connection stiffness

n = shape factor

θ_o = reference plastic rotation, calculated as $\theta_o = M_{ult}/R_{ki}$

M_{ult} = ultimate moment capacity of the connection

As shown in Figure 4, the shape factor n defines the shape of the $M-\theta$ curve for a given set of R_{ki} and M_{ult} values.

Equations for R_{ki} , M_{ult} and n for connections made from angle sections have been derived (Kishi et al. [41]). With reference to a typical connection angle shown in Figure 5, if we define

g_c = distance between heel of angle to center of first line of fasteners

k = gauge distance from heel of angle to top of fillet

t = angle thickness

W = nut width

and denote

d = beam depth

l = angle length

$I_o = t^3/12$ = geometrical moment of inertia

$M_o = \sigma_y t^2/4$ = moment per unit length of angle

σ_y = yield stress of angle

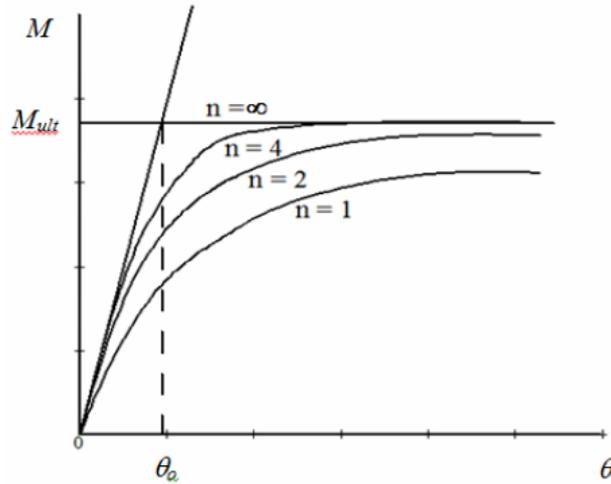


Figure 4. Three-parameter Power Connection Model

as well as with the use of the following non-dimensional parameters

$$\beta = g_c/l, \quad \gamma = l/t, \quad \delta = d/t, \quad \kappa = k/t, \quad \omega = W/t, \quad \rho = t_w/t_t \quad (2a-f)$$

where subscripts t and w denote the top angle and the web angle, respectively, the equations for the three parameters R_{ki} , M_{ult} and n are given as follows.

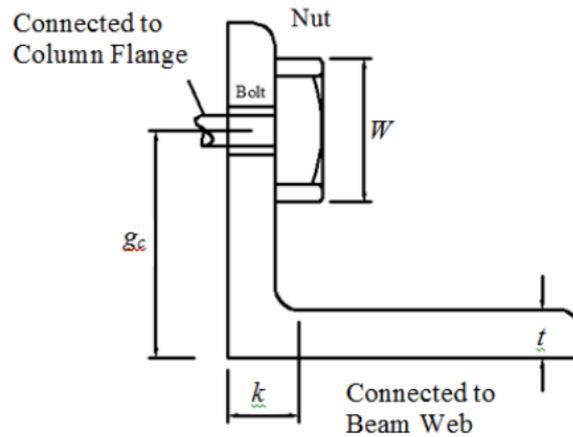


Figure 5. Symbols used for an Angle Connection

3.1 Initial Connection Stiffness R_{ki}

The equation for the initial connection stiffness is

$$\frac{R_{ki}}{EI_{ot}} = \frac{R_{kits}}{EI_{ot}} + \frac{R_{kiw}}{EI_{ot}} \quad (3)$$

where R_{kits}/EI_{ot} and R_{kiw}/EI_{ot} are the non-dimensional top and seat angle and web angle stiffness contributions to the initial connection stiffness, respectively. They are given by

$$\frac{R_{kits}}{EI_{ot}} = \frac{3(1 + \delta_t)^2}{\beta_t'[\gamma_t^2(\beta_t')^2 + 0.78]} \quad (4)$$

$$\frac{R_{kiw}}{EI_{ot}} = \frac{3\rho(1 + \delta_t)^2}{2\beta'_w[\gamma_w^2(\beta'_w)^2 + 0.78]} \quad (5)$$

in which

$$EI_{ot} = 29000 \cdot \frac{t_t^3}{12} \quad (6)$$

$$\beta'_t = \beta_t - \frac{1}{2\gamma_t} (1 + \omega_t) \quad (7)$$

$$\beta'_w = \beta_w - \frac{1}{2\gamma_w} (1 + \omega_w) \quad (8)$$

3.2 Ultimate Connection Moment Capacity M_{ult}

The equation for the ultimate connection moment capacity is

$$\frac{M_{ult}}{M_{ot}t_t} = \frac{M_{uts}}{M_{ot}t_t} + \frac{M_{uw}}{M_{ot}t_t} \quad (9)$$

where $M_{uts}/M_{ot}t_t$ and $M_{uw}/M_{ot}t_t$ are the top and seat angle and web angle moment contributions to the ultimate connection moment capacity, respectively. They are given by

$$\frac{M_{uts}}{M_{ot}t_t} = \gamma_t \left\{ 1 + \xi_t \left[1 + \beta_t^* + 2(\kappa_t + \delta_t) \right] \right\} \quad (10)$$

$$\frac{M_{uw}}{M_{ot}t_t} = \gamma_w (1 + \xi_w) \rho^3 \left[\frac{\gamma_w (\xi_w - 1)}{3(\xi_w + 1)} + \delta_w + \frac{1}{\rho} \right] \quad (11)$$

where

$$M_{ot} = \frac{\sigma_y t_t^2}{4} \quad (12)$$

and ξ_t and ξ_w are to be evaluated from the following fourth-order equations

$$\xi_t^4 + \beta_t^* \xi_t - 1 = 0 \quad (13)$$

$$\xi_w^4 + \beta_w^* \xi_w - 1 = 0 \quad (14)$$

where

$$\beta_t^* = \beta_t \gamma_t - \kappa_t \quad (15)$$

$$\beta_w^* = \beta_w \gamma_w - \kappa_w \quad (16)$$

with β'_t computed from Eq. 7.

It should be noted that even though Eqs. 3 and 9 were derived for top and seat angle connections with double web angles, they can be simplified for top and seat angles without the web angles by setting the second term equal zero, or for double web angle connections by setting the first term equal zero, or for single web angle connections by setting the first term equal zero and dividing the second term by 2.

3.3 Shape Parameter n

The equations for the shape parameter are given as follows.

For top and seat angle connections with the web angles:

$$n = 1.398 \log_{10} \theta_o + 4.631 \geq 0.827 \quad (17a)$$

For top and seat angle connections without the web angles:

$$n = 2.003 \log_{10} \theta_o + 6.070 \geq 0.302 \quad (17b)$$

For double web angle connections:

$$n = 1.322 \log_{10} \theta_o + 3.952 \geq 0.573 \quad (17c)$$

For single web angle connections:

$$n = 0.520 \log_{10} \theta_o + 2.291 \geq 0.695 \quad (17d)$$

Because of some uncertainties in test setups and data collections, the loss of accuracy during the curve-fitting process, and the unavoidable variability of one connection to another, a resistance factor of 0.9 as recommended by Christopher and Bjorhovde [42] will be used in the proposed method of PR frame design.

4. AISC DIRECT ANALYSIS METHOD

The direct analysis method is a method recommended by AISC for use in the stability analysis and design of frame structures. It has the following features:

1. A second-order elastic analysis that explicitly accounts for the P - δ and P - Δ effects is to be used to determine the required strengths. However, it is permissible to neglect the P - δ effect in the analysis (but not in design) if (a) the gravity loads are supported primarily by nominally vertical columns, walls or frames, (b) the ratio of maximum second-order drift to first-order drift both calculated using adjusted stiffness (see Items 3 and 4 below) for all stories is ≤ 1.7 , and (c) no more than one-third of the total gravity loads on the structure is supported by columns that are part of the moment-resisting frame in the direction of translation being considered. In lieu of a second-order analysis, the use of a first-order analysis in conjunction with the use of moment magnification (i.e., B_1 and B_2) factors is permitted.
2. The effect of initial imperfections in the form of column out-of-plumbness is to be considered in the analysis by including them directly in the structural model. The magnitude of the initial displacements shall be the maximum amount considered in the design, and the pattern of initial displacements shall be such that it provides the greatest destabilizing effect. Alternatively, for structures that support gravity loads primarily through nominally vertical columns, walls or frames, the effect of initial imperfections can be represented using notional loads. In this approach, notional lateral loads of magnitude $0.002 \sum P_i$ (where $\sum P_i$ is the factored gravity loads acting on story i) distributed over the story

in the same proportion as the gravity loads are to be applied to the story for all gravity-only load combinations. However, for frames where the ratio of second-order to first-order drift (both computed using the adjusted stiffness discussed in Items 3 and 4 below) exceeds 1.7, these notional lateral loads have to be applied to *all* load combinations in the analysis.

3. A reduced stiffness equal to $0.8E$ is to be used for members whose flexural or axial stiffness is considered to contribute to the lateral stability of the frame. Conservatively, the use of $0.8E$ for *all* members is permitted. This stiffness reduction is used to account for the effects of member imperfections and inelasticity, as well as uncertainty in determining the member strength and stiffness.
4. If $P_u/P_y > 0.5$ (where P_u is the required axial compressive strength and $P_y = F_y A$ is the yield load), a second stiffness reduction factor, $\tau = 4(P_u/P_y)[1 - (P_u/P_y)]$ is to be applied to the moment of inertia I of all flexural members that contribute to the lateral stiffness of the frame. In lieu of using τ , an increase of the notional from $0.002\sum P_i$ to $0.003\sum P_i$ can be used in all load combinations.
5. Regardless of the end conditions of the member, an effective length factor of $K=1$ is to be used to compute the design compressive strength P_n of the member.

5. PROPOSED PR FRAME DESIGN APPROACH

The steps proposed for the design of PR frames are as follows:

1. For each semi-rigid connection, assume a value of $\bar{R}_k = R_{kb}L/EI$, where R_{kb} is the effective linearized connection stiffness under factored gravity loads. It is obtained as the slope of a line drawn from the origin to the intersection point of the PR connection $M-\theta$ curve and a beam line (see Figure 6). The beam line is defined by the equation

$$M = \frac{w_u L^2}{12} - \frac{2EI}{L} \theta \quad (18)$$

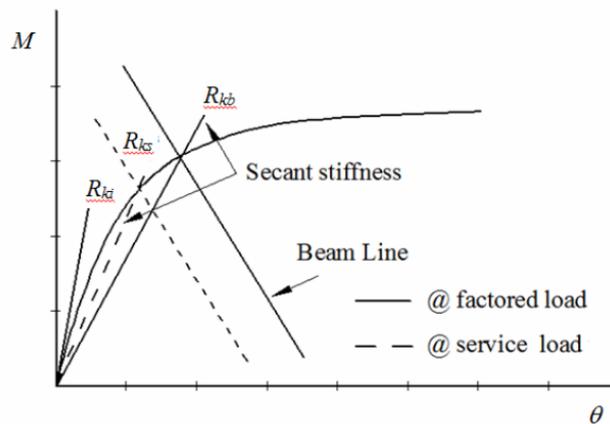


Figure 6. Determination of R_{kb}

where w_u is the uniform factored gravity load on the beam. The above equation relates the end moment M and rotation θ of a prismatic beam subject to a uniformly distributed load applied over its entire length. Note that if the connection is fully-rigid, $\theta=0$ and so $M=w_u L^2/12$, which is the fixed-end moment of the beam. On the other hand, if the connection is ideally-pinned, $M=0$ and so $\theta=w_u L^3/24EI$, which is the end rotation of a simply-supported beam.

2. Calculate the beam moments M_{neg} and M_{pos} for the assumed value of \bar{R}_k using the following equations (Chen and Lui [34]).

$$M_{neg} = \frac{2\bar{R}_{kb}}{3(\bar{R}_{kb} + 2)} M_{ss} \quad (19)$$

$$M_{pos} = \frac{\bar{R}_{kb} + 6}{3(\bar{R}_{kb} + 2)} M_{ss} \quad (20)$$

where $M_{ss} = w_u L^2 / 8$ is the uniformly loaded simply-supported beam moment.

3. Select trial beam section based on the calculated M_{pos} and M_{neg} values.
4. Select the type of connection (e.g., a top and seat angle connection with double web angles) and choose a family of connections (3 to 4 etc.) as trials.
5. Determine the $M-\theta$ curve parameters (M_{ult} , R_{ki} , and n) for each connection using the procedure outlined in Section 3.
6. Construct a family of $M-\theta$ curves using Eq. 1, and superimpose on them a beam-line computed using Eq. (18)
7. Select an appropriate connection by determining the R_{kb} values for the family of connections and choose the one that has a $\bar{R}_k (= R_{kb}L/EI)$ value closest to the one assumed in Step 1.
8. Determine R_{ks} as shown in Figure 6 and calculate $R_{ks}L/EI$ for the selected connection at service load to check that it falls in the PR connection range of $2 < R_{ks}L/EI < 20$.
9. Select the trial column section based on the factored axial load and moment.
10. For unbraced frames, calculate the loading stiffness R_{kL} using the equation

$$R_{kL} = \frac{M_n - M_g}{\theta_n - \theta_g} \quad (21)$$

where, in reference to Figure 7, M_n is the nominal moment strength of the connection that corresponds to $\theta_n = 0.02$ radian (AISC [22]) denoted as point b in the figure, and M_g and θ_g are the moment and rotation of the connection under factored gravity loads denoted as point a.

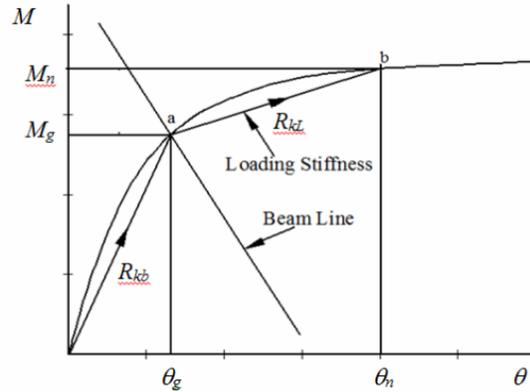
11. Analyze the PR frame using the appropriate connection stiffness R_{kb} , R_{kL} or R_{ki} in conjunction with the Direct Analysis Method for the two cases described below.

Case 1, the non-sway frame condition for gravity load combinations only:

Element Stiffness - Column: $EI_e = 0.8 \tau EI$
 - Beams: $EI_e = 0.8 EI$
 - Connections: $R_k = 0.9 R_{kb}$

Applied Loads - gravity load combinations, plus a notional load of $0.002 P_i$ applied to each column at each floor level.

- Perform a *second-order elastic analysis* to determine the internal forces.
- Check the beam-column, beam, and connection strength.
- If the above conditions are satisfied, proceed further. Otherwise, revise the sections and/or connections until the conditions are satisfied.

Figure 7. Determination of R_{kL}

Case 2, the sway frame condition for gravity + lateral load combinations:

Step 1 - Gravity load step:

Element Stiffness - Column: $EI_e = 0.8 \tau EI$

- Beams: $EI_e = 0.8 EI$

- Connections: $R_k = 0.9 R_{kb}$

Applied Loads - Factored gravity loads only without any notional loads

- Perform a *second-order elastic analysis* to determine the internal forces due to the applied loads.

Step 2 - Lateral load step:

Element Stiffness - Column: $EI_e = 0.8 \tau EI$

- Beams: $EI_e = 0.8 EI$

- Connections: $R_k = 0.9 R_{ki}$ (Windward Side)

$R_k = 0.9 R_{kL}$ (Leeward Side)

Applied Loads - Concentrated vertical loads applied at beam-column joints producing the column axial forces associated with *Step 1*
 - Lateral load combinations plus a notional load of $0.002 P_i$ applied to each column at each floor level

- Perform a *second-order elastic analysis* to determine the internal forces due to above applied loads
- Superimpose the moments from the gravity (step 1) and lateral (step 2) load steps. Check the beam-column, beam and connection strengths. If not satisfied, revise the sections and/or connections of the structure.

6. DESIGN EXAMPLES

In this section, design examples will be given to show how the proposed PR frame design procedure presented in the preceding section can be applied. The second-order analysis is performed using the software MASTAN2 (McGuire et al. [43]).

The first example (a portal frame) is used to demonstrate the basic loading/unloading behavior of PR connections in an unbraced frame when one end of a beam/girder is connected to a single column. It also serves to show the effect of PR connections on the columns. The second example (a 2-story unbraced frame) is used to demonstrate how the moments can be distributed to the columns above and below the beam to which the columns are attached. In both examples, only top and seat angle with double web angle connections will be considered. This is because the normalized stiffness \bar{R}_k for this type of connections often falls in the semi-rigid range of $2 < \bar{R}_k < 20$, and so they are commonly used in PR frames. In addition, the following assumptions have been used:

- All beam sections are compact (i.e., the sections are capable of developing the full plastic moment)
- All column sections are non-slender elements (i.e., local buckling is not of concern)
- All sections are oriented with their webs in the plane of the applied loads
- The frames are braced against out-of-plane deflection at the beam-column joints only
- The beams are non-composite (i.e., they are not connected to a concrete slab)
- Wind loads are applied as concentrated loads at windward beam-column joints.
- All beam-column connections satisfy the AISC connection design requirements
- All beam and column sections are W-shapes made from A992 steel with $F_y = 50$ ksi (345 MPa) and $E = 29000$ ksi (200 GPa).

Example 1 – Design of a PR Portal Frame

The PR frame shown in Figure 8 is to be designed. The frame is subjected to the loads shown. Beam deflection under service live load is limited to $L/360$, where L is the beam length; and frame drift should not exceed $H/200$, where H is the story height. To start the design, assume $\bar{R}_k = 6$ for the connections.

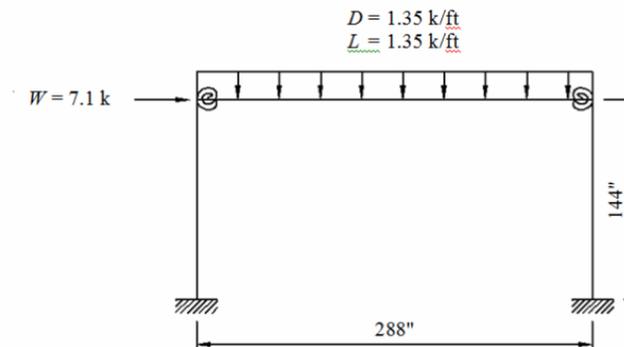


Figure 8. An Unbraced Portal PR Frame (1 k = 4.45 kN, 1 ft = 12" = 0.305 m)

Solution:

Load Combinations: The load combinations (AISC [22], ASCE [44]) that most likely will control the design are given in Table 1.

Table 1. Applied loads (1 k/in. = 175 kN/m, 1 kip = 4.45 kN)

Load Case	Load Combinations	w_u (k/in.) (Gravity)	p_u (kips) (Lateral)
1	$1.2D + 1.6L$	0.315	-
2	$1.2D + 0.5L + 1.0W$	0.191	7.1

Beam Section Selection: For load case 1, the frame does not undergo any sideways movement. Use Eqs. (19) and (20) with the assumed $\bar{R}_k = 6$ to estimate M_{neg} and M_{pos} for the preliminary sizing of the beam.

$$M_{neg} = \frac{2\bar{R}_{kb}}{3(\bar{R}_{kb} + 2)} M_{ss} = 1633 \text{ k-in.}$$

$$M_{pos} = \frac{\bar{R}_{kb} + 6}{3(\bar{R}_{kb} + 2)} M_{ss} = 1633 \text{ k-in.}$$

Using these moments as a guide, a W14×53 is selected as the trial section. The flexural capacity $\phi_b M_n$ of this section is 2462 k-in. (278 kN-m), which exceeds the required flexural strength M_u ($= M_{neg}$ or M_{pos}) = 1633 k-in. (185 kN-m).

Connection Selection: If top and seat angle with double web angle connections are to be used for the PR frame, a possible set of candidate connections are summarized in Table 2. In the table, the thickness of the top and seat angle is denoted as C (5/8), C (3/4), etc.

Table 2. Details of PR Connections (1 in. = 25.4 mm)

Connection	Bolt diameter d (in.)	Nut Width W (in.)	Top and Seat Angles $2L6 \times 4 \times t_t$				Web Angles $2L4 \times 3.5 \times \frac{1}{2}$		
			t_t (in.)	l_t (in.)	g_{ct} (in.)	k_t (in.)	l_w (in.)	g_{cw} (in.)	k_w (in.)
C (5/8)	7/8	1-7/16	5/8	7	3	1-1/8	8	3	11/16
C (3/4)	7/8	1-7/16	3/4	7	3	1-1/4	8	3	11/16
C (7/8)	7/8	1-7/16	7/8	7	3	1-3/8	8	3	11/16

Calculations for R_{ki} , M_{ult} , and n : Using the above angle data and Eqs. (3), (9) and (17a), the three connection parameters are calculated and summarized in Table 3.

Table 3. Connection Parameters (1 k-in. = 0.113 kN-m)

Connection	$R_{ki} (\times 10^3)$ (k-in./rad)	M_{ult} (k-in.)	n
C (5/8)	411	1907	1.37
C (3/4)	690	2435	1.20
C (7/8)	1112	2973	1.03

Beam-Line Diagrams with $M-\theta$ Curves: The intersection of the $M-\theta$ curve and the beam line represents the expected connection moment and deformation under gravity loads. Using Eq. (1) for the $M-\theta$ curves and Eq. (18) for the beam lines, one can generate Figure 9 for the three connections and the two load cases given in Table 1. The beam lines for the two load cases are labeled (BL-LC(I) and BL-LC(II), respectively, in the figure. From this figure, R_{kb} for each connection can be determined as the slope of a line drawn from the origin to the intersection point of the connection's $M-\theta$ curve and the respective beam line. The calculated secant stiffness for each connection under Load Case 1 R_{kb1} is given in Table 4.

Table 4. Connection Stiffness (1 k-in. = 0.113 kN-m)

Connection	Connection Stiffness, R_{kbI} ($\times 10^3$) (k-in./rad)	$\bar{R}_k = R_{kbI}L/EI$
C (5/8)	435	8
C (3/4)	314	5.8
C (7/8)	202	3.7

Connection C (3/4) is selected because its \bar{R}_k value is closest to the initially assumed value of 6. Once the connection has been selected, its R_{ks} and R_{kL} values can now be determined. By subjecting the frame to the service gravity load ($D+L$), $R_{ks}L/EI$ is computed to be 7.51, which falls in the PR connection range. From Figure 9, the design strength ($\phi_b M_n$ (0.02 rad)) of the connection is determined to be 1987 k-in. (225 kN-m). Using the data presented in Figure 9 and the schematics shown in Figure 7, R_{kb} and R_{kL} for the two load cases are computed and given in Table 5.

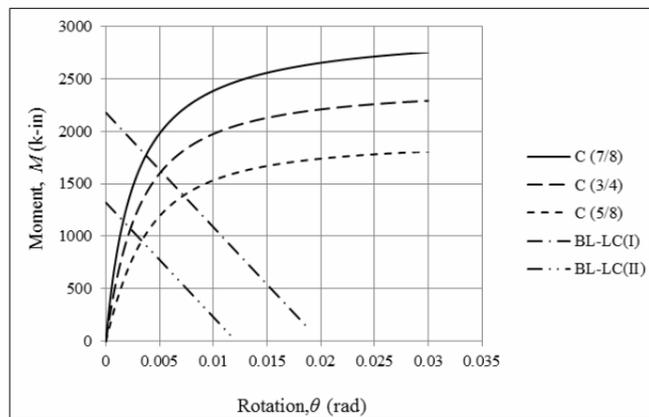
Figure 9. M - θ curves and beam lines (1 k-in. = 0.113 kN-m)

Table 5. Linearized Connection Stiffness (1 k-in./rad = 0.113 kN-m/rad)

Beam Line for Load Case (BL-LC)	Linearized Connection Stiffness	
	R_{kb} ($\times 10^3$) (k-in./rad)	R_{kL} ($\times 10^3$) (k-in./rad)
1	314	-
2	467	64

Column Section Selection: A W10 \times 33 section is selected based on the axial load (P_u) and moment (M_{neg}) acting on the beam-column joint due to load case 1. A 12-ft long W10 \times 33 section has an axial capacity ($\phi_c P_n$) of 292 kips (1300 kN) and bending moment capacity ($\phi_b M_n$) of 1530 k-in. (173 kN-m).

Applying the Direct Analysis Approach: The analysis is carried out for the non-sway and sway frame conditions as outlined in Section 5.

Case 1 – the non-sway frame condition for gravity loads only

The parameters used for this analysis are summarized in Table 6.

Table 6. Case 1 Analysis Parameters (1 k = 4.45 kN, 1 in. = 25.4 mm)

Element Stiffness			Applied Loads	
Columns	Beams	Connections	Load	Notional Load
$EI_e=0.8\tau EI$ (k-in. ²)	$EI_e=0.8EI$ (k-in. ²)	$0.9R_{kbl}$ (k-in./rad)	$1.2D+1.6L$ (k/in.)	$0.002P_i$ (kips)
3.97×10^6	12.6×10^6	282×10^3	0.315	0.091

(τ is taken as 1 since the axial load P_u in each column for this load case is less than $0.5P_y$.)

Using the above parameters, a second-order elastic analysis is performed to determine the internal forces in each member. The results are shown in Table 7. Also shown in the table are the results obtained from a 2nd-order inelastic analysis (i.e., instead of using the direct analysis approach outlined in Section 4, the frame is analyzed using 2nd-order inelastic analysis technique) as well as the results of the unity check obtained using the appropriate beam-column interaction (H1-1a or H1-1b) or beam equation given in AISC [22]. The unity check is satisfied if the value computed is <1. As can be seen, the results obtained using the direct analysis approach is slightly conservative when compared to those obtained from a 2nd-order inelastic analysis.

Table 7. Analysis Results for Case 1 with Unity Check (1 kip = 4.45 kN, 1 k-in. = 0.113 kN-m)

Case 1						
Element	Axial Force (kips)		Moment (k-in.)		Unity Check	
	Proposed Method	2 nd -order Inelastic Analysis	Proposed Method	2 nd -order Inelastic Analysis	Proposed Method	2 nd -order Inelastic Analysis
Left Column	45.3	45.3	1029	992	0.75	0.73
Right Column	45.4	45.4	1040	1004	0.76	0.73
Beam	-	-	2246	2280	0.91	0.93

Case 2 – the sway frame condition for gravity + wind loads

The analysis parameters for this load case are given below.

- (i) Gravity load step: $1.2D+0.5L$ (see Table 8)

Table 8. Case 2 Gravity Load Analysis Parameters (1 k = 4.45 kN, 1 in. = 25.4 mm)

Element Stiffness			Applied Loads	
Columns	Beams	Connections	Loads	Notional Load
$EI_e=0.8\tau EI$ (k-in. ²)	$EI_e=0.8EI$ (k-in. ²)	$0.9R_{kbl}$ (k-in./rad)	$1.2D+0.5L$ (k/in.)	$0.002P_i$ (kips)
3.97×10^6	12.6×10^6	420×10^3	0.191	n.a

(ii) Lateral load step: $1.0W$ (see Table 9)

Table 9. Case 2 Lateral Load Analysis Parameters (1 k = 4.45 kN, 1 in = 25.4 mm)

Element Stiffness				Applied Loads		
Column $EI_e=0.8\tau EI$ (k-in. ²)	Beam $EI_e=0.8EI$ (k-in. ²)	Connections		Loads		Notional Load $0.002P_i$ (kips)
		$0.9R_{ki}$ (k-in./rad)	$0.9R_{kL}$ (k-in./rad)	Column Load (kips)	Wind $1.0W$ (kips)	
3.97×10^6	12.6×10^6	621×10^3	58×10^3	27.5	7.1	0.055

Using the analysis parameters given in Tables 8 and 9, a second-order elastic analysis is performed individually for each load step to determine the internal forces in the structure. The results from these two analyses are then superimposed and presented in Table 10. Note that while the final moments are determined by superimposing the moment results from both the gravity and gravity+wind load steps, the final axial loads are determined only from the gravity+wind load step because the axial load effect from the gravity load step is already accounted for by subjecting the columns to the concentrated joint loads. Note also that the notional loads (representing non-verticality) are only applied in the gravity+wind load step to ensure that the second-order effects associated with the non-verticality are accounted for only once. Also shown in the table are results from a 2nd-order inelastic analysis and results of the unity check.

Table 10. Analysis Results for Case 2 with Unity Check (1 kip = 4.45 kN, 1 k-in. = 0.113 kN-m)

Case 2						
Element	Axial Force (kips)		Moment (k-in.)		Unity Check	
	Proposed Method	2 nd -order Inelastic Analysis	Proposed Method	2 nd -order Inelastic Analysis	Proposed Method	2 nd -order Inelastic Analysis
Left Column	26	26	404	392	0.33	0.3
Right Column	29	29	809	778	0.57	0.56
Beam	-	-	1382	1406	0.56	0.57

Finally, the service live load deflection of the beam and the wind drift of the frame are calculated to be 0.41 in. (10.4 mm) and 0.32 in. (8.1 mm), respectively. Since they are less than the allowable values of $L/360 = 0.8$ in. (20.3 mm) and $H/200 = 0.72$ in. (18.3 mm), the design is considered satisfactory.

Example 2 - Design of a Two-Story Unbraced PR Frame

The PR frame subjected to the applied loads shown in Figure 10 is to be designed. The live load beam deflection and the wind induced interstory frame drift are to be limited to $L/360$ and $H/200$, respectively. To start the design, assume $\bar{R}_k = 8$ and 10 for the 1st and 2nd story connections, respectively.

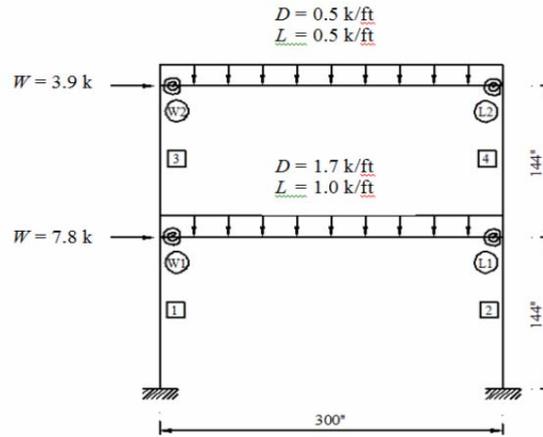


Figure 10. A Two-story Unbraced PR Frame (1 k = 4.45 kN, 1 ft = 12” = 0.305 m)

Solution:

The load combinations used for the design are given in Table 11. By using Load Case 1, Eqs. 19 and 20, and the assumed \bar{R}_k values, M_{neg} and M_{pos} are calculated and shown in Table 12. From these moment values, a W12×50 section ($\phi_b M_n = 1903$ k-in. or 215 kN-m) and a W10×33 section ($\phi_b M_n = 912$ k-in. or 103 kN-m) are selected for the beams of the first and second stories, respectively.

Table 11. Applied Loads (1 k/in. = 175 kN/m, 1 kip = 4.45 kN)

Load Case	Load Combinations	w_{u1} (k/in.) (Gravity)	w_{u2} (k/in.) (Gravity)	p_{u1} (kips) (Lateral)	p_{u2} (kips) (Lateral)
1	$1.2D + 1.6L$	0.303	0.117	-	-
2	$1.2D + 0.5L + 1.0W$	0.212	0.071	7.8	3.9

Table 12. Non-sway Beam Moments for Beam Section Selection (1 k-in. = 0.113 kN-m)

Story	M_{neg} (k-in.)	M_{pos} (k-in.)
1 st	1818	1590
2 nd	731	585

A set of possible top and seat angle with double web angle connections that can be used for the PR frame is given in Table 13. Using the values shown in this table, the connection parameters for the three-parameter model are calculated using Eqs. (3), (9), (17a) and are summarized in Table 14. The $M-\theta$ curves of these connections are plotted in Figures 11 and 12 together with the beam lines computed from Eq. (18) for the two load cases. From these figures, the secant stiffness for load case 1 R_{kbl} can be determined and are presented in Table 15. Connections C (3/4) and C (1/2) are selected because their \bar{R}_k values are closest to their respective assumed values. Using the methodology depicted schematically in Figure 7, the linearized connection stiffness are determined for the two load cases and shown in Table 16.

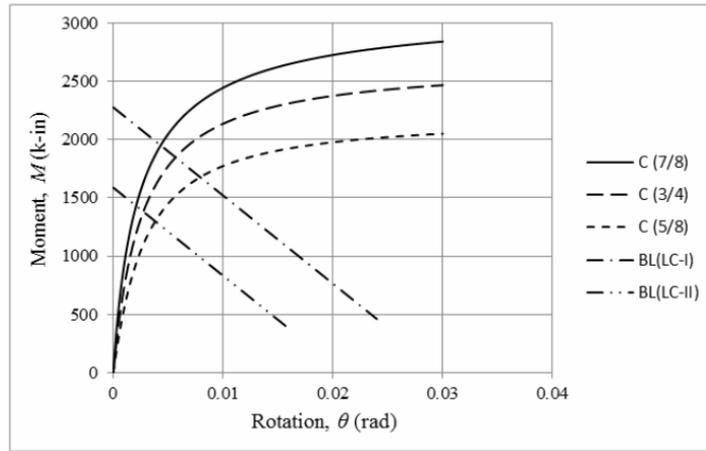


Figure 11. First Story $M-\theta$ Curves and Beam Lines (1 k-in. = 0.113 kN-m)

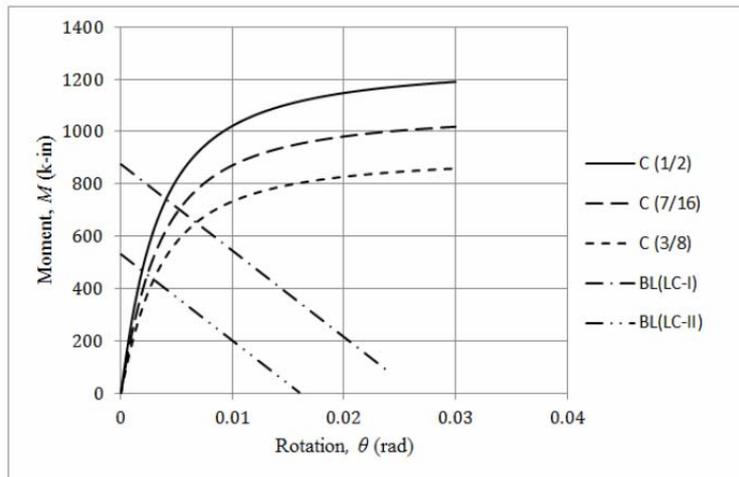


Figure 12. Second Story $M-\theta$ Curves and Beam Lines (1 k-in. = 0.113 kN-m)

Table 13. PR Connection Details (1 in. = 25.4 mm)

1 st Story									
Connection	Bolt Diameter d (in.)	Nut Width W (in.)	Top and Seat Angles $2L6 \times 4 \times t_t$				Web Angles $2L4 \times 4 \times 5/8$		
			t_t (in.)	l_t (in.)	g_{ct} (in.)	k_t (in.)	l_w (in.)	g_{cw} (in.)	k_w (in.)
C (5/8)	7/8	1-7/16	5/8	7	2.75	1-1/8	8	2.5	1
C (3/4)	7/8	1-7/16	3/4	7	2.75	1-1/4	8	2.5	1
C (7/8)	7/8	1-7/16	7/8	7	2.75	1-3/8	8	2.5	1
2 nd Story									
Connection	Bolt Diameter d (in.)	Nut Width W (in.)	Top and Seat Angles $2L6 \times 4 \times t_t$				Web Angles $2L4 \times 3.5 \times 1/2$		
			t_t (in.)	l_t (in.)	g_{ct} (in.)	k_t (in.)	l_w (in.)	g_{cw} (in.)	k_w (in.)
C (3/8)	7/8	1-7/16	3/8	7	2.5	7/8	8	2.25	7/8
C (7/16)	7/8	1-7/16	7/16	7	2.5	15/16	8	2.25	7/8
C (1/2)	7/8	1-7/16	1/2	7	2.5	1	8	2.25	7/8

Table 14. Connection Parameters (1 k-in. = 0.113 kN-m)

Connection	$R_{ki} (\times 10^3)$ (kips-in./rad)	M_{ult} (kips-in.)	n
1st Story			
C (5/8)	686	2187	1.41
C (3/4)	1010	2663	1.03
C (7/8)	1497	3140	0.89
2nd Story			
C (3/8)	212	908	1.32
C (7/16)	258	1077	1.31
C (1/2)	322	1261	1.27

Table 15. Connection Stiffness (1 k-in. = 0.113 kN-m)

Connection	Connection Stiffness, $R_{kbl} (\times 10^3)$ (k-in./rad)	$\bar{R}_k = R_{kbl}L/EI$
1st story		
C (5/8)	209	5.52
C (3/4)	325	8.6
C (7/8)	453	12
2nd story		
C (3/8)	92	5.6
C (7/16)	142	8.6
C (1/2)	183	11

Table 16. Linearized Connection Stiffness(1 k-in./rad = 0.113 kN-m/rad)

Beam Line for Load Case (BL-LC)	Linearized Connection Stiffness			
	$R_{kb} (\times 10^3)$ (k-in./rad)		$R_{kL} (\times 10^3)$ (k-in./rad)	
	1 st Story	2 nd story	1 st Story	2 nd story
1	325	183	-	-
2	572	246	66	38

Based on P_u and M_{neg} calculated for the gravity load case, a W10×33 ($\phi_c P_n=292$ kips or 1300 kN, $\phi_b M_n=1530$ k-in. or 173 kN-m) is selected as a trial section for the columns. Because each end of the 1st story beam is connected to two columns, a distribution factor based on the flexural stiffness of the columns above and below the beam as given by Eqs. (22a and 22b) can be used to apportion M_{neg} to the columns.

$$M_{above} = \frac{\left(\frac{I}{L}\right)_{above}}{\left(\frac{I}{L}\right)_{above} + \left(\frac{I}{L}\right)_{below}} M_{neg} \quad (22a)$$

$$M_{below} = \frac{\left(\frac{I}{L}\right)_{below}}{\left(\frac{I}{L}\right)_{above} + \left(\frac{I}{L}\right)_{below}} M_{neg} \quad (22b)$$

Using the analysis parameters computed and given in Tables 17 and 18, a direct analysis as outlined in Section 4 is carried out for the two load cases. The results together with the 2nd-order inelastic analysis and unity check results are given in Table 19. As can be seen, good correlation is observed between the direct analysis results and those obtained using 2nd-order inelastic analysis. The unity check is also satisfied for all members.

Table 17. Case 1 Analysis Parameters (1 k = 4.45 kN, 1 in. = 25.4 mm)

Story	Element Stiffness			Applied Loads	
	Columns $EI_e=0.8\tau EI$ (k-in. ²)	Beam $EI_e=0.8EI$ (k-in. ²)	Connections $0.9R_{kb1}$ (k-in./rad)	Gravity Load $1.2D+1.6L$ (k/in.)	Notional Load $0.002P_i$ (kips)
1 st	3.97×10^6	9.07×10^6	292×10^3	0.303	0.091
2 nd	3.97×10^6	3.97×10^6	165×10^3	0.117	0.035

Table 18. Case 2 Analysis Parameters(1 k = 4.45 kN, 1 in. = 25.4 mm)

Story	Element Stiffness			Applied Loads			
	Gravity load step						
	Columns $EI_e=0.8\tau EI$ (k-in. ²)	Beam $EI_e=0.8EI$ (k-in. ²)	Connections $0.9R_{kb2}$ (k-in./rad)	Load $1.2D+0.5L$ (k/in.)	N.L $0.002P_i$ (kips)		
1 st	3.97×10^6	9.07×10^6	515×10^3	0.212	-		
2 nd	3.97×10^6	3.97×10^6	222×10^3	0.071	-		
	Lateral load step						
	Columns $EI_e=0.8\tau EI$ (k-in. ²)	Beam $EI_e=0.8EI$ (k-in. ²)	Connections		Load		Notional Load $0.002 P_i$ (kips)
			$0.9R_{ki}$ (k-in./rad)	$0.9R_{kL}$ (k-in./rad)	Column Load (kips)	Wind $1.0W$ (kips)	
1 st	3.97×10^6	9.07×10^6	909×10^3	59×10^3	31.8	7.8	0.064
2 nd	3.97×10^6	3.97×10^6	290×10^3	34×10^3	10.6	3.9	0.021

The service live load deflections of the 1st and 2nd story beams are computed to be 0.37 in. (9.4 mm) and 0.44 in. (11.1 mm), respectively, and the wind induced interstory drifts of the frame are calculated to be 0.62 in. (15.7 mm) for the 1st story and 0.68 in. (17.2 mm) for the 2nd story. They are within the allowable values of $L/360=0.833$ in. (21.2 mm) for beam deflection and $H/200=0.72$ in. (18.2 mm) for interstory drift, and so the design is considered satisfactory.

Table 19. Analysis results for Cases 1 and 2 with Unity Check
(1 kip = 4.45 kN, 1 k-in. = 0.113 kN/m)

Load Case	Element	Axial Force (kips)		Moment (k-in.)		Unity Check	
		Proposed method	2 nd -order Inelastic Analysis	Proposed method	2 nd -order Inelastic Analysis	Proposed method	2 nd -order Inelastic Analysis
1	Col 1	63	63	685	662	0.61	0.60
	Col 2	63	63	699	676	0.62	0.61
	Col 3	17.5	17.5	866	836	0.60	0.58
	Col 4	17.6	17.6	870	840	0.60	0.58
	1 st story Beam	-	-	1843	1897	0.97	0.99
	2 nd story Beam	-	-	637	656	0.69	0.72
2	Col 1	38	39	349	347	0.33	0.3
	Col 2	46	46	826	809	0.64	0.61
	Col 3	9.5	9.5	440	423	0.30	0.3
	Col 4	12	12	687	665	0.47	0.45
	1 st story Beam	-	-	1350	1388	0.71	0.73
	2 nd story Beam	-	-	412	423	0.45	0.46

7. SUMMARY AND CONCLUSIONS

A procedure for the design of partially-restrained (PR) frames is proposed. In a PR frame, the connections used to connect the beams and columns are semi-rigid in nature. According to AISC [22], a connection is considered semi-rigid if the ratio of its secant stiffness under service load R_{ks} to the beam stiffness EI/L falls in the range $2 < R_{ks}L/EI < 20$. The proposed procedure makes use of the three-parameter connection model to describe the nonlinear connection moment-rotation ($M-\theta$) behavior and the direct analysis approach to perform the frame analysis. To proceed with the design, a connection stiffness R_{kb} defined as the slope of a line drawn from the origin to the intersection point of the nonlinear $M-\theta$ curve and the beam line constructed for the factored gravity load case under consideration is used. Loading and unloading behavior of the PR connections are accounted for by the use of two different stiffness R_{kL} and R_{ki} ; the former is obtained from linearization of the $M-\theta$ curve and the latter is obtained from the three-parameter connection model.

Trial beam sections are selected based on the beam moments M_{pos} and M_{neg} calculated using a target value of $\bar{R}_k = R_{kb}L/EI$ for the connections under factored gravity loads. The connections are then selected based on the proximity of their \bar{R}_k values to those of the target values. Once the connections are selected, trial column (beam-column to be exact) sections can be selected based on the approximate P_u and M_u values computed from the applied gravity loads and M_{neg} of the adjoining connections, respectively.

Examples of PR frames designed using top and seat with web angle connections were given to demonstrate the steps involved in applying the proposed method in PR frame design. It was shown that despite its simplicity, the proposed approach yields good results when compared to a 2nd-order inelastic analysis.

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