BEHAVIOUR AND DESIGN OF FABRICATED HIGH STRENGTH STEEL COLUMNS SUBJECTED TO BIAXIAL BENDING

PART II: ANALYSIS AND DESIGN CODES

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ABSTRACT: This paper considers the behaviour of hollow and concrete filled steel columns fabricated with high strength structural steel plate and subjected to biaxial bending. The columns considered in this paper are fabricated from high strength quenched and tempered structural steel plate of nominal yield stress of 690 N/mm². The principal parameter that has been considered in the test program is the slenderness of the component plates. The results of the experiments have been compared with existing codes for the consideration of steel and steel-concrete composite columns under biaxial loading conditions. These include the Eurocode 3 and 4 document, (British Standards Institution, 1992 and 1994), the Australian AS4100 design provisions for steel (Standards Australia, 1998) and American Standard LRFD (American Institute of Steel Construction, 2000). The experiments did reveal the beneficial effect of the use of concrete infill on delaying the onset of local buckling and thereby the post-local buckling behaviour of the columns. A numerical model to consider the coupled effect of local and global buckling under biaxial conditions has been developed herein. This model considers the behaviour of short and slender concrete filled high strength steel columns under biaxial loading incorporating material and geometric non-linearities. For this analysis the equilibrium of the member is investigated in the deformed state, using the idealised stress-strain relationships for both materials steel and concrete, considering the elastic and plastic ranges. The paper concludes with a suggested design approach and makes suggestions for further research.

Keywords: biaxial loading, buckling, composite structures, high strength steel, local buckling, steel columns, steel construction, tall buildings, welded columns

1. INTRODUCTION

The experiments presented in part I of this paper are quite novel and help to elucidate the behaviour of both hollow and concrete filled high strength steel sections subjected to biaxial loading conditions. However, for design purposes, it is extremely important to have a routine design procedure which is representative, consistent and conservative in the prediction of the ultimate strength. The scope of existing standards of design is limited to the design of structural members fabricated from steel with yield stress not exceeding 460 N/mm².

The purpose of this paper is few fold. Firstly the paper compares existing slenderness criteria for composite and hollow steel sections. Secondly the paper presents a model for considering interaction buckling for composite and hollow steel columns subjected to biaxial bending. Thirdly the paper compares the existing code methods for fabricated column under biaxial bending with experimental results. Finally then the paper draws a broad conclusion of this and the accompanying paper.

2. CODE LIMITING SLENDERNESS

This section provides a comparison of the plate slenderness limit (λ_e) based on international code standards AS4100 [1], Eurocode 3 [2], Eurocode 4 [3] and AISC-LRFD [4]. The non dimensional plate element slenderness (λ_e) defined in AS4100 was used and incorporated in other code specifications to enable the comparisons between the different codes of practice.

2.1 Elastic Local Buckling Formulae and AS4100

For a compact steel plate ($\lambda_e < \lambda_{ey}$), the plates will buckle above the yield stress. In contrast, a slender plate ($\lambda_e \geq \lambda_{ey}$) will buckle first prior to reaching the yield stress. This stress at which buckling takes place is known as the elastic local buckling stress (σ_{ol}). The elastic local buckling stress (σ_{ol}) is calculated using the well known approach

$$\sigma_{ol} = \frac{k\pi^2 E}{12(1-\nu^2) \left(\frac{b}{t}\right)^2} \quad (1)$$

which has been elucidated for concrete filled section [5]. When any of the plate elements have slenderness limits that exceed the specified limits, the strength is governed by local buckling. When applied to a hollow box section, the strength of a cross-section is given by:

$$P_{eff} = f_y A_e \quad (2)$$

where A_e is the effective area of the cross-section, used to account for the effective width (b_e) of the plate. The plate slenderness limit (λ_e) defined in AS4100 is given by:

$$\lambda_e = \frac{b}{t} \sqrt{\frac{f_y}{250}} \quad (3)$$

The local buckling coefficient (k) was taken to be 4.0 for local buckling of plates in hollow steel members. From the tensile tests, the elastic modulus was determined as $E=207875\text{N/mm}^2$, and the Poisson's ratio $\nu=0.3$. The effective width principle is used to determine the post-buckling behaviour of the plate components, and it is given by the following expression,

$$\frac{b_e}{b} = \alpha_{rg} \sqrt{\frac{\sigma_{ol}}{f_y}} \quad (4)$$

Using the local buckling formula in Eq. (1), the limiting effective width ratio (b_e/b) of the plate element is reduced to,

$$\frac{b_e}{b} \leq 0.063 \frac{b/t}{\lambda_e} \alpha_{rg} \sqrt{\sigma_{ol}} \quad (5)$$

where the yield slenderness limit (λ_{ey}) of the plate element may be obtained when (b_e/b) is equal to unity. To determine the plate yield slenderness limit, λ_{ey} may be expressed in the following expression:

$$\lambda_{ey} = \frac{2\alpha_{rg}}{\sqrt{1000}} \sqrt{\frac{k\pi^2 E}{12(1-\nu^2)}} \quad (6)$$

The yield stress was taken as 760 N/mm², which was obtained from the compressive stub-column tests. Hence, the plate slenderness limit ($\lambda_{ey}=34.9$) for high strength structural steel is determined by taking $\alpha_{rg}=0.65$ Uy [2001] to account for residual stress and geometric imperfections specifically for welded steel box sections.

2.2 Eurocode 3

In Eurocode 3, the effective width principle is applicable for cross sections exceeding the b/t limits for Class 3 sections. The effective width of the plate element (b_e) is obtained using the following expression,

$$b_e = \rho b \quad (7)$$

where

$$\rho = \frac{\bar{\lambda}_p - 0.22}{\bar{\lambda}_p^2} \quad (8)$$

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{ol}}} \quad (9)$$

The non-dimensional slenderness ($\bar{\lambda}_p$) of the plate element may be defined in terms of the b/t limit of the cross section by using Eq. (9). Hence, using the local buckling stress formula given in Eq. (1) and the same material properties as the abovementioned, the non-dimensional slenderness ($\bar{\lambda}_p$) of the plate element is obtained as,

$$\bar{\lambda}_p = 1.05 \frac{b}{t} \sqrt{\frac{f_y}{Ek}} \quad (10)$$

The limiting effective width ratio (b_e/b) in Eurocode 3 may then be obtained using Eq. (10), which is expressed as a function of the slenderness limit (λ_e) given in AS4100. Thus, the local buckling slenderness limit for high strength structural steel is 36.7.

$$\frac{b_e}{b} \leq \frac{54.7}{\lambda_e} \left(1 - \frac{12.1}{\lambda_e} \right) \quad (11)$$

2.3 Eurocode 4

In the Eurocode 4 specification, there is no recommendation for the use of the effective width principle for a composite section, in which the b/t limit exceeds the Class 3 sections or also known as a slender section. In the Eurocode 4 specification, the slenderness limit ($b/t \leq 52\epsilon$) for concrete-filled hollow sections can be expressed in the form of the AS4100 yield slenderness limit (λ_e), given in Eq. (12).

$$\lambda_e \leq \frac{104\varepsilon f_y}{\sqrt{1000}} \quad (12)$$

$$\leq 50.4$$

where

$$\varepsilon = \sqrt{\frac{235}{f_y}}$$

The determined slenderness limit (λ_e) can be used to obtain the local buckling coefficient (k) by using the plate slenderness limit give in Eq. (6). In this case, the local buckling coefficient (k) for composite columns is determined as 8. Based on previous studies by Uy and Bradford [6] on local buckling of plates in composite members, the local buckling coefficient (k) was derived as 10.31. This value was used in the formulation of the slenderness limit for composite members. The limit ratio can be defined using $k=10.31$, and thus the yield slenderness limit (λ_e) can be increased to 57.2.

2.4 AISC-LRFD

The American structural steel limit state code AISC-LRFD (2000) expresses the effective width of the plates in the form:

$$b_e = 1.91t \sqrt{\frac{E}{F_y}} \left[1 - \frac{0.381}{b/t} \sqrt{\frac{E}{F_y}} \right] \quad (13)$$

where F_y is the steel yield stress in AISC-LRFD. The effective area (A_e) ratio is stated as:

$$A_e = QA \quad (14)$$

which makes use of the historical Q-factor concept used in American structural steel specifications. In the AISC-LRFD code provision, the Q-factor is expressed in the form $Q_s Q_a$. For cross-sections with stiffened plate elements, $Q_s=1.0$ and hence, the effective area (A_e) is reduced to $Q_a A$. The value of Q is used in determining the strength of the cross-section, which is highlighted in the following sections. The effective width ratio (b_e/b) may be expressed as function of the slenderness limit (λ_e) defined in AS4100, and is obtained by the following expression. Hence, the slenderness limit for hollow columns in AISC-LRFD is found to be 39.9.

$$\frac{b_e}{b} \leq \frac{55.1\lambda_e - 604.5}{\lambda_e^2} \quad (15)$$

3. MODELING

For short columns under biaxial loading a general method has been developed to define the ultimate strength analysis of composite members incorporating local and post local buckling. The method allows points on the failure surface of composite members to be determined. In the method the concrete core and steel plates are subdivided into small elements. The assumption of a plane distribution of strain over the section is considered. True stress strain behaviour for both the materials of steel and concrete are applied. The slenderness ratios for the component steel plates are

defined and elastic local buckling factors are determined respectively for each plate considering the applied stress gradient as shown in Figure 1. The applied stresses are calculated in accordance with the depth and the angle of the neutral axis as illustrated in Figure 2. Internal forces are formed through integration of the stresses then the angle and eccentricity of the applied force are calculated. These main procedures are outlined in the following.

- a) provide input data including section dimensions, material properties and angle of the neutral axis
- b) discretise the section into (N) small elements
- c) define the coordinates of the centroid of each element $X(I,J)$, $Y(I,J)$
- d) assign a value to the depth of the neutral axis Z_n
- e) define the initial slenderness ratio of the component steel plates in accordance with the depth of the neutral axis
- f) define the initial ultimate concrete strain in accordance with the defined initial slenderness ratio of the component steel plates
- g) define the elastic local buckling factors respectively for each plate considering the applied stress gradient
- h) define the real slenderness ratio of the component steel plates in accordance with the calculated elastic local buckling factors
- i) calculate the effective width for each component plate if any of the component steel plates is slender
- j) define the ultimate concrete strain and determine if the concrete is confined or unconfined in accordance with the plate slenderness ratio
- k) repeat steps (g, h, i) for the defined ultimate concrete strain
- l) define the strain at the centroid of each element
- m) calculate the axial force (N_s) and the moment (M_s) in the steel ignoring the ineffective area
- n) calculate the axial force (N_c) and the moment (M_c) in the concrete taking into consideration if it is confined or unconfined
- o) calculate the resultant axial force (N) and moment (M_x) in the section
- p) calculate the eccentricity of the axial force to the centroid of the section E_{XC} , E_{YC}
- q) define the angle (Ψ) of the applied axial force
- r) repeat the above procedures for another value of Z_n

For slender columns the deflected shape is assumed to be a cosine curve. The angle θ of the deflection plan with the x-axis and the maximum deflection at the mid height section define the deflected shape. Consideration of the deformations in the section at mid-height shows that the neutral axis of the strains is perpendicular to the deflection plane. From the eccentricity at inclination ψ_0 to the x-axis plus the deflection at mid height at inclination θ the total resultant eccentricity of the applied force can be defined. This eccentricity can be defined also from the above mentioned procedures. By employing a search technique, which equates the two total eccentricities and the two angles of the applied load within an allowable tolerance the load moment curve of the column can be obtained. This is illustrated in Figure 3.

The stress-strain behaviour for high strength steel has been idealised; so that the steel behaves in an elastic linear fashion followed by a plastic range until the ultimate stress is reached. This was piecewise linearised to reflect the measured values from experiments.

Two concrete stress-strain relationships have been used. The first is an unconfined model employing the CEB-FIP model [7]. This model is useful in modeling the concrete filled steel columns when local buckling takes place prematurely and thus the steel does not adequately confine the concrete. The second one is a confined model developed by Tomi [8]. This model is useful to account for the effects of concrete confinement as experienced in concrete filled steel columns with compact plates.

Figure 1. Buckling Coefficient vs Stress Gradient for Composite Section

Figure 2. Section Strain Distribution Under Biaxial Loading

Figure 3. Model of the Deflected Shape of Column and Section under Biaxial Loading

Local buckling takes place when there are localized cross sectional deformations and the buckled shape is of the order of the depth or width of the section. When a composite section has a thin-walled steel section, which is able to buckle locally prior to yield, then a reduction in the strength of the steel section must be made. Local buckling of concrete filled steel columns is restrained by the concrete core and the steel component may only buckle away from the concrete. This results in a higher load carrying capacity compared with that of the bare steel columns. The local buckling stress does not represent failure and a significant redistribution of stress occurs after local buckling.

Local buckling of the component plates is calculated using the approach of Uy [5], which accounts for the effect of the restraint offered by the concrete. The elastic local buckling stress is determined by defining the buckling coefficient in accordance with the stress gradient, Figure 1. This allows one to determine the inelastic local buckling strength or post-buckling behaviour. The post-local buckling behaviour is best characterised using an effective width principle which has been calibrated by Uy [5, 9, 10].

For post-buckling in the inelastic range a novel method, which considers strain energy of the component plates is used. The inelastic local buckling strain is calculated by utilising the results of an inelastic finite strip analysis. The post buckling behaviour is then characterised by using the inelastic effective width, which is determined from the inelastic buckling strain and the yield strain. This has been explained in the paper published by Mursi and Uy [11].

3.1 Comparisons with Model

To verify the suitability of the model the calculated results are compared with the experimental data for short and slender columns which has been fully detailed in part I of this paper Mursi and Uy [12]. The above mentioned model has been used to define the ultimate strength capacity of steel and composite members under biaxial bending incorporating local and post local buckling. This enables one to determine the points on the failure surface according to the angle and the eccentricity of the applied load. In the experiments conducted the load has been applied at an equal eccentricity about each axis which defines that the angle of the applied load is 45 degrees.

Figures 4 to 13 illustrate the model and the experimental results. For short columns the ultimate strength of the section capacity incorporating local buckling under biaxial load has been determined. For slender columns the load-moment curve has been generated considering local and overall buckling. The model can define accurately the points where local buckling takes place in the component steel plates.

Short composite columns SH-C160 and SH-C110 which, in terms of slenderness, have compact composite sections show that the experimental results are close enough to the confined model of the section capacity. This fact illustrates that the confinement effect of concrete can be considered in compact composite sections. The results are illustrated in Figures 5 and 7. Furthermore, short hollow columns SH-H260, SH-H210 and SH-H110, which are classified as slender sections, show that the experimental results are higher than the model section capacity. This can be related to the high value of the factor $\alpha_{rg} = 0.65$, which is used in the model to account for residual stress and geometric imperfections.

Figure 4. Comparison Between Model and Experimental Result for Hollow Short and Slender Column (SH – H110 & SL - H110)
The Biaxial Load Applied at 45 Degree with Eccentricity 10 mm in each Direction

Figure 5. Comparison of model and experimental result for composite short and slender column (SH – C110 & SL – C110)

The biaxial load applied at 45 degree with eccentricity 10 mm in each direction

Figure 6. Comparison of Model and Experimental Result for Hollow Short and Slender Column (SH – H160 & SL – H160)

The Biaxial Load Applied at 45 Degree with Eccentricity 15 mm in each Direction

Figure 7. Comparison of Model and Experimental Result for Composite Short and Slender Column (SH – C160 & SL – C160)
The Biaxial Load Applied at 45 Degree with Eccentricity 15 mm in each Direction

Figure 8. Comparison of Model and Experimental Result for Hollow Short and Slender Column with (SH – H210 & SL – H210)
The Biaxial Load Applied at 45 Degree with Eccentricity 20 mm in each Direction

Figure 9. Comparison of Model and Experimental Result for Composite Short and Slender Column (SH – C210 & SL – C210)

The Biaxial Load Applied at 45 Degree with Eccentricity 20 mm in each Direction

Figure 10. Comparison of Model and Experimental Result for Hollow Short and Slender Column (SH – H210 & SL40 – H210)

The Biaxial Load Applied at 45 Degree with Eccentricity 40 mm in each Direction

Figure 11. Comparison of Model and Experimental Result for Composite Short and Slender Column (SH – H210 & SL40 – C210)
The Biaxial Load Applied at 45 Degree with Eccentricity 40 mm in each Direction

Figure 12. Comparison of Model and Experimental Result for Hollow Short and Slender Column (SH – H260 & SL – H260)

The Biaxial Load Applied at 45 Degree with Eccentricity 25 mm in each Direction

Figure 13. Comparison of Model and Experimental Result for Composite Short and Slender Column (SH – C260 & SL – C260)
The Biaxial Load Applied at 45 Degree with Eccentricity 25 mm in each Direction

Slender composite columns SL-C260, SL-C210 and SL40-C210 illustrate a higher load capacity, which exceeds the unconfined section behaviour and is closer to that of the confined behaviour of concrete. This suggests that the higher slenderness limit should be used in determining the slenderness of a composite section. The other slender composite column SL-C160, for which the section is compact, reflects also a high load capacity, and the failure load is very close to the confined section capacity, as illustrated in Figure 7. Whilst the slender composite column SL-C110 failed by overall buckling, the section failure occurred in the post peak range, where the load moment curve intersects with the confined section capacity curve and This is illustrated in Figure 5. Slender hollow columns SL-H260, SL-H210, SL40-H210, SL-H160, SL-H110, which all contain slender cross sections, failed by a combination of local and overall buckling. It is worth noting that all of them failed at a higher load than the section strength curve which suggests that slenderness effects were minimal.

4. DESIGN METHODS

Various codes of practice have specified design approaches for hollow and concrete-filled steel columns. A comparative study is undertaken herein which reviews the design codes with experimental results for evaluating the ultimate strength of hollow and composite columns subjected to biaxial loading. The effects of local buckling were also considered. In the following sections, a review of design approaches for hollow and composite columns under biaxial bending is presented mainly based on AS4100 (Standards Australia, 1998), Eurocode 3 and 4, (British Standards Institution, 1992 and 1994) and LRFD (American Institute of Steel Construction, 2000).

4.1 Australian Standard AS 4100

4.1.1 Design rules for hollow steel columns under biaxial bending

4.1.1.1 Section capacity of hollow steel columns under biaxial bending

Local Buckling

In the Australian Steel Structures Code AS 4100 [1], the slenderness limit (λ_e) of a steel plate element is given as follows:

$$\lambda_e = \frac{b}{t} \sqrt{\frac{f_y}{250}} \quad (16)$$

where

b = the plate width of the steel element

t = the thickness of the plate

f_y = the yield stress of the steel in N/mm²

For a heavily welded plate element supported along both longitudinal edges, the yield slenderness limit is given as $\lambda_{ey} = 35$.

Section Capacity under Combined Compression and Biaxial Bending

The design approach for the section capacity of compact rectangular and square hollow sections subjected to combined axial compression and biaxial bending is given as follows:

$$\frac{N}{N_s} + \frac{M_x}{M_{sx}} + \frac{M_y}{M_{sy}} \leq 1 \quad (17)$$

where:

N = the design axial force

N_s = the nominal section axial load capacity

M_{sx}, M_{sy} = the nominal section moment capacity about the major x and y -axis respectively

The axial load capacity may be obtained as,

$$N_s = k_f A_g f_y \quad (18)$$

where

f_y = the yield stress of the steel in N/mm^2

k_f = the form factor

$$= \frac{A_e}{A_g} \quad (19)$$

The effective area A_e is obtained by adding the areas of individual elements and taking into account the effective width of the plate elements. The effective width (b_e) of a flat plate element can be determined from the value of the element slenderness (λ_e) and the element yield slenderness limit (λ_{ey}). The effective width (b_e) for a flat element is expressed as,

$$b_e = b \left(\frac{\lambda_{ey}}{\lambda_e} \right) \leq b \quad (20)$$

where

b = the clear width of the element between the faces of the supporting plate elements

$\lambda_{ey} = 35$ (Heavily welded plate element)

The nominal section moment capacity about the two axes can be expressed as,

$$M_{sx} = f_y Z_{ex} \quad (21)$$

$$M_{sy} = f_y Z_{ey} \quad (22)$$

i. For compact sections $\lambda_e \leq \lambda_{ep}$,

where $\lambda_{ep} = 30$,

Z_e = the effective section modulus = the lesser of ($S, 1.5Z$)

Z = the elastic section modulus

S = the plastic section modulus

f_y = the yield stress of steel in N/mm^2

ii. For non-compact sections $\lambda_{ep} \leq \lambda_e \leq \lambda_{ey}$,

where $\lambda_{ep} = 30$ and $\lambda_{ey} = 35$,

$$Z_e = Z + \left[\left(\frac{\lambda_{ey} - \lambda_e}{\lambda_{ey} - \lambda_{ep}} \right) (Z_c - Z) \right] \quad (23)$$

Z_c = the effective section modulus for a compact section

iii. For slender sections $\lambda_e \geq \lambda_{ey}$,

where $\lambda_{ey} = 35$,

$$Z_e = Z \left(\frac{\lambda_{ey}}{\lambda_e} \right) \quad (24)$$

4.1.1.2 Member capacity under combined compression and biaxial bending

According to AS 4100, the member design approach for a hollow steel member under combined compression and biaxial bending is based on the following nonlinear interaction formula

$$\left(\frac{M_x}{M_{cx}} \right)^{1.4} + \left(\frac{M_y}{M_{by}} \right)^{1.4} \leq 1 \quad (25)$$

where

M_{cx} = the lesser of the nominal in-plane member moment capacity (M_{ix}) about the major x -axis and the nominal out-of-plane member moment capacity (M_{ox}) about the major x -axis

M_{ox} = the nominal out-of-plane member moment capacity about the major x -axis

$$M_{ox} = M_{bx} \left(1 - \frac{N}{N_{cy}} \right) \quad (26)$$

N = the applied compressive load

$$M_{bx} = \text{the nominal member moment capacity} = \alpha_m \alpha_s M_s \leq M_s \quad (27)$$

$$M_s = \text{the nominal section capacity} = f_y Z_e \quad (28)$$

f_y = the yield strength of steel, N/mm²

$$N_c = \text{the nominal member capacity} = \alpha_c N_s \leq N_s \quad (29)$$

$$\begin{aligned} N_s &= \text{the nominal section capacity} \\ &= k_f A_g f_y \end{aligned} \quad (30)$$

$$\begin{aligned} k_f &= \text{the reduction factor due to effective area of the steel cross-section} \\ &= \frac{A_e}{A_g} \end{aligned} \quad (31)$$

A_g = the gross cross-sectional area of the steel

$$\begin{aligned} b_e &= \text{the effective width of the steel} \\ &= b \left(\frac{\lambda_{ey}}{\lambda_e} \right) \end{aligned} \quad (32)$$

$$\begin{aligned} \alpha_c &= \text{the member slenderness reduction factor} \\ &= \xi \left(1 - \sqrt{1 - \left(\frac{90}{\xi \lambda} \right)^2} \right) \end{aligned} \quad (33)$$

$$\begin{aligned} \xi &= \text{the compression member factor} \\ &= \frac{\left(\frac{\lambda}{90} \right)^2 + 1 + \eta}{2 \left(\frac{\lambda}{90} \right)^2} \end{aligned} \quad (34)$$

$$\begin{aligned} \lambda &= \text{the slenderness ratio} \\ &= \lambda_n + \alpha_a \alpha_b \end{aligned} \quad (35)$$

$$\begin{aligned} \eta &= \text{the compression member imperfection factor} \\ &= 0.00326 (\lambda - 13.5) \geq 0 \end{aligned} \quad (36)$$

$$\begin{aligned} \lambda_n &= \text{the modified compression member slenderness} \\ &= \frac{l_e}{r} \sqrt{k_f} \sqrt{\frac{f_y}{250}} \end{aligned} \quad (37)$$

$$\begin{aligned} \alpha_a &= \text{the compression member factor} \\ &= \frac{2100(\lambda_n - 13.5)}{\lambda_n^2 - 15.3\lambda_n + 2050} \end{aligned} \quad (38)$$

For $k_f < 1.0$

$$\alpha_b = 0 \quad (\text{Welded steel box sections})$$

Second Order Effects in Slender Members of Columns

The load-carrying capacity of biaxially loaded columns will be reduced due to second order effects in addition to its member slenderness. The moment magnifier approach is used for braced (non-sway) columns to take into account second order effects. The amount of magnified column moment depends on the effective length of the column. If the member slenderness is high, the magnification effect will be high.

The moment magnifier (δ_b) is determined from Eq. (39). At each value of the applied load N , δ_b is evaluated. The second order moments may be expressed as $\delta_b M$.

$$\begin{aligned} \delta_b &= \text{the moment amplification factor} \\ &= \frac{C_m}{1 - \frac{N}{N_{cr}}} \geq 1 \end{aligned} \quad (39)$$

$$\begin{aligned} N_{cr} &= \text{the elastic critical buckling load for a long column} \\ &= \frac{\pi^2 E_s I}{l_e^2} \end{aligned} \quad (40)$$

$$\begin{aligned} C_m &= \text{the factor for unequal moments} \\ &= 0.66 - 0.44\beta_m \end{aligned} \quad (41)$$

$$\begin{aligned} \beta_m &= \text{the ratio of end moment to fixed end moment} \\ &= -1 \end{aligned} \quad (42)$$

The value of β is equal to -1 as the column is bent into single curvature by an equal magnitude of axial force acting at an equal eccentric distance away from the plastic centroid.

Using Eq. (25), the applied moment taking into account second order effects, $\delta_b M$, can be calculated from $\delta_b N e$ using Eq. (25), where N and e are the applied load and eccentricity of the axial force applied to the column, respectively.

4.1.2 Suggested approach for composite columns under biaxial bending

The design approach for a composite member subjected to biaxial bending can be undertaken by using the Australian steel standard rules. To make this possible the concrete area should be transformed to an equivalent area of steel section. The section and the column load capacity is then determined using the AS4100 method for steel structures. The transformation is described as follows,

The width of transformed area from concrete to steel (b_t) can be calculated from,

$$b_t = \frac{b}{n} \quad (43)$$

where

$$\begin{aligned} n &= \text{the modular ratio} \\ &= \frac{E_s}{E_c} \end{aligned} \quad (44)$$

E_s = the elastic modulus of the steel

$$\begin{aligned} E_c &= \text{the elastic modulus of the concrete} \\ &= 0.043 \rho^{1.5} \sqrt{f_c} \end{aligned} \quad (45)$$

f_c = the compressive strength of cylinder

The total cross-sectional area of steel and transformed steel can be determined by using the transformed width shown above. The second moment of area of the steel and transformed steel can also be obtained by taking into account the transformed width. Then, by following the above mentioned AS4100 design specifications for steel columns, the section and member capacities under biaxial bending can be determined.

4.2 Eurocode 3 Design Rules for Hollow Steel Columns

4.2.1 Local buckling

The width-to-thickness ratio for the steel section of hollow members is given by the following equation:

$$\frac{b_l}{t} \leq 42\varepsilon \quad (46)$$

where

b_l = the greater overall dimensions of the section = $b - 3t$

t = the thickness of the steel plate

$$\varepsilon = \sqrt{\frac{235}{f_y}}$$

f_y = the yield strength of the steel in N/mm²

According to Eurocode 3, the above width-to-thickness ratio expression is only applicable to Classes 1, 2 and 3 structural elements. When the b/t exceeds this limit, the influence of local buckling on the strength of the structural members needs to be taken into account. In this case, the cross-section shall be classified as Class 4, as stated from the code.

4.2.2 Section capacity under combined compression and biaxial bending

In Eurocode 3, the design approach for a hollow steel section under combined compression and biaxial bending is based on the following linear interaction formula:

$$\frac{N_{Sd}}{N_{b,Rd}} + \frac{M_{y,Sd}}{M_{y,Rd}} + \frac{M_{z,Sd}}{M_{z,Rd}} \leq 1 \quad (47)$$

For a thin-walled square or rectangular steel section subjected to combined axial compression and biaxial bending, it shall satisfy the following relation:

$$\frac{N_{Sd}}{A_{eff} \frac{f_y}{\gamma_M}} + \frac{M_{y,Sd}}{W_{eff,y} \frac{f_y}{\gamma_M}} + \frac{M_{z,Sd}}{W_{eff,z} \frac{f_y}{\gamma_M}} \leq 1 \quad (48)$$

where

W_{eff} = the effective section modulus about the axis of bending

A_{eff} = the effective area of the section

f_y = the yield strength of the steel in N/mm²

$$\begin{aligned} b_{eff} &= \rho d \\ &= \rho (d - 3t) \end{aligned} \quad (49)$$

ρ = the plate buckling reduction factor

$$\rho = \frac{\bar{\lambda}_p - 0.22}{\bar{\lambda}_p^2} \quad (50)$$

$\bar{\lambda}_p$ = the non-dimensional slenderness of a flat plate

$$\begin{aligned} &= \sqrt{\frac{f_y}{f_E}} \\ &= \frac{b/t}{28.4\epsilon\sqrt{k_\sigma}} \end{aligned} \quad (51)$$

f_y = the yield strength of the steel

f_E = the critical plate buckling stress

k_σ = the buckling factor
= 4

$$\epsilon = \sqrt{\frac{235}{f_y}} \quad (52)$$

4.2.3 Member capacity under combined compression and biaxial bending

In Eurocode 3, the member design approach for a hollow steel column under combined compression and biaxial bending is based on the following linear interaction formula:

$$\frac{N_{Sd}}{N_{b,Rd}} + k_y \frac{M_{y,Sd}}{M_{y,Rd}} + k_z \frac{M_{z,Sd}}{M_{z,Rd}} \leq 1 \quad (53)$$

For class 4 steel cross-sections subjected to combined compression and biaxial bending, it shall satisfy the following relation:

$$\frac{N_{Sd}}{A_{eff} \frac{f_y}{\gamma_M}} + k_y \frac{M_{y,Sd}}{W_{eff,y} \frac{f_y}{\gamma_M}} + k_z \frac{M_{z,Sd}}{W_{eff,z} \frac{f_y}{\gamma_M}} \leq 1 \quad (54)$$

where

k_y, k_z = the amplification coefficients for a column to account for second order effects

$$= 1 - \frac{\mu_y N_{Sd}}{\chi_y N_{pl}} \leq 1.5 \quad (55)$$

$$= 1 - \frac{\mu_z N_{Sd}}{\chi_z N_{pl}} \leq 1.5 \quad (56)$$

μ_y, μ_z = the coefficients used for a column

$$= \bar{\lambda}_y (2\beta_{M,y} - 4) + \left(\frac{W_{pl,y}}{W_{el,y}} - 1 \right) \leq 0.9 \quad (57)$$

$$= \bar{\lambda}_z (2\beta_{M,z} - 4) + \left(\frac{W_{pl,z}}{W_{el,z}} - 1 \right) \leq 0.9 \quad (58)$$

in which χ_y and χ_z are reduction factors about the y and z -axis respectively. β_y and β_z are the equivalent uniform moment factors. $M_{y,Sd}$ and $M_{z,Sd}$ are the design moment capacities about the y and z -axis respectively. N_{Sd} is the design value of axial compression.

The design buckling resistance capacity of the member is given by,

$$N_{b,Rd} = \chi \frac{A_s f_y}{\gamma_m} \quad (59)$$

where

χ = the reduction factor for buckling curves

A_s = the cross-sectional area of steel

f_y = the yield strength of steel

γ_M = the partial safety factor for resistance = 1.1

The reduction factor for buckling χ is given by,

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \leq 1 \quad (60)$$

$$\phi = 0.5 \left[1 + \alpha \left(\bar{\lambda} - 0.2 \right) + \bar{\lambda}^2 \right] \quad (61)$$

where

α = the imperfection factor for the corresponding buckling curve
 = 0.21 (Curve a)
 = 0.34 (Curve b)
 = 0.49 (Curve c)
 = 0.76 (Curve d)

The non-dimensional slenderness $\bar{\lambda}$ is given by,

$$\bar{\lambda} = \frac{\lambda}{\lambda_E} \quad (62)$$

where

$$\begin{aligned} \lambda &= \text{the slenderness of a column} \\ &= \frac{l_b}{i} \end{aligned} \quad (63)$$

l_b = the effective buckling length

i = the radius of gyration

$$\begin{aligned} \lambda_E &= \text{the Eulerian slenderness} \\ &= \pi \sqrt{\frac{E}{f_y}} \end{aligned} \quad (64)$$

$$E = 210000 \text{ N/mm}^2$$

4.3 Eurocode 4 Design Rules for Composite Column

The slenderness limitation for Eurocode 4 for concrete filled box sections is given by:

$$\frac{b_1}{t} \leq 52\varepsilon \quad (65)$$

where

b_1 = the greater overall dimensions of the section = $b - 3t$

$$\varepsilon = \sqrt{\frac{235}{f_y}}$$

The M - N interaction curve shows the section capacity of a column subjected to bending about the principal axis. The rigid plastic analysis with rectangular stress block for the steel and concrete element is adopted in Eurocode 4. It assumes the steel or concrete to be fully yielded or not stressed at all. The curve shown in Figure 14 presents the 4 interaction points A, B, C and D.

In Eurocode 4, for the design of a composite column under combined compression and biaxial bending, the axial resistance of a column must be determined for each axis. Checks need to be carried out for combined compression and uniaxial bending to determine which axis is more critical.

$$M_{v,Sd} \leq 0.9 \mu_y M_{ply,Rd} \quad (66)$$

$$M_{z,Sd} \leq 0.9 \mu_z M_{plz,Rd} \quad (67)$$

$$\frac{M_{y,Sd}}{\mu_y M_{ply,Rd}} + \frac{M_{z,Sd}}{\mu_z M_{plz,Rd}} \leq 1 \quad (68)$$

where

μ_y, μ_z = the moment resistance ratio about the y and z -axis, respectively

$M_{ply,Rd}, M_{plyz,Rd}$ = the plastic bending resistance of the relevant plane of bending

$M_{y,Sd}, M_{z,Sd}$ = the design bending moment about y and z -axis including second order effects and imperfections

Effective Elastic Flexural Stiffness of Cross-Sections $(EI)_e$

For short term loading, the flexural stiffness of the cross section is given by,

$$(EI)_e = E_s I_s + 0.8 E_c I_c \quad (69)$$

where

$$E_c = \frac{E_{cm}}{\gamma_c} \quad (70)$$

E_c = the secant modulus of the concrete

$$= 9.5(f_c + 8)^{0.33} \quad (71)$$

γ_c = the partial safety factor = 1.35

E_s = the modulus of the steel

I_s, I_c = the second moment of inertia of the steel and the concrete, respectively

The resistance of a column member in axial compression N_{sd}

$$N_{sd} = \chi N_{pl,Rd} \quad (72)$$

where

$N_{pl,Rd}$ = the resistance of the cross-section

$$= \frac{A_s f_y}{1.1} + \frac{A_c f_c}{1.5} \quad (73)$$

A_s = the cross-sectional area of steel

A_c = the cross-sectional area of concrete

f_y = the yield strength of steel

f_c = the yield strength of concrete

χ = the reduction coefficient for strength of column member and can be determined from the slenderness λ and strength curve

The reduction factor for buckling χ is given by,

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \leq 1 \quad (74)$$

$$\phi = 0.5 \left[1 + \alpha \left(\bar{\lambda} - 0.2 \right) + \bar{\lambda}^2 \right] \quad (75)$$

where

α = the imperfection factor for the corresponding buckling curve
= 0.21 (Curve a)

The relative slenderness $\bar{\lambda}$ is given by,

$$\bar{\lambda} = \sqrt{\frac{N_{pl,Rd}}{N_{cr}}} \quad (76)$$

N_{cr} = the elastic critical load

$$= \frac{\pi^2 (EI)_e}{l_e^2} \quad (77)$$

4.4 AISC–LRFD Design Rules for Hollow Steel Columns

4.4.1 Local buckling

In the Load and Resistance Factor Design Specification of the American Institute of Steel Construction (AISC-LRFD), the minimum width-to-thickness-ratio of uniformly compressed stiffened elements is given as follows:

$$\frac{b}{t} \geq 1.40 \sqrt{\frac{E}{F_y}} \text{ (Slender)} \quad (78)$$

$$\frac{b}{t} \leq 1.12 \sqrt{\frac{E}{F_y}} \text{ (Compact)} \quad (79)$$

where

E = the modulus of elasticity of steel in ksi or N/mm²

F_y = the yield stress of the steel in ksi or (N/mm²)

4.4.2 Member capacity under combined compression and biaxial bending

In the Load and Resistance Factor Design Specification of the American Institute of Steel Construction (AISC-LRFD), the interaction equation for doubly symmetric hollow members in combined axial compression and biaxial bending is given by the following Eqs.

For $\frac{P_u}{P_n} \geq 0.2$,

$$\frac{P_u}{P_n} + \frac{8}{9} \left(\frac{M_{ux}}{M_{nx}} + \frac{M_{uy}}{M_{ny}} \right) \leq 1 \quad (80)$$

For $\frac{P_u}{P_n} < 0.2$,

$$\frac{P_u}{2P_n} + \left(\frac{M_{ux}}{M_{nx}} + \frac{M_{uy}}{M_{ny}} \right) \leq 1 \quad (81)$$

where

M_{ux}, M_{uy} = the required moment capacity for bending component in the plane of x and y -axis respectively

M_{nx}, M_{ny} = the nominal bending strength with bending only in the plane of x and y -axis respectively

P_u = the required axial capacity

P_n = the nominal compressive strength in kips or (N) = $A_g F_{cr}$

A_g = the gross cross-sectional area

Figure 15 illustrates the type of interaction curve for a doubly symmetric hollow section under combined axial load and bending about one axis using the LRFD method.

Figure 15. Interaction Diagram for Combined Axial Load and Bending about One Axis Only Based on AISC - LRFD

i. For $\lambda \leq \lambda_p$,

where $\lambda_p = 1.12 \sqrt{\frac{E}{F_y}}$,

$$M_n = M_p = F_y Z \quad (82)$$

Z = the plastic section modulus in LRFD

F_y = the yield stress of steel in ksi or (N/mm²)

ii. For $\lambda_p \leq \lambda \leq \lambda_r$,

where $\lambda_p = 1.12 \sqrt{\frac{E}{F_y}}$ and $\lambda_r = 1.40 \sqrt{\frac{E}{F_y}}$,

$$M_n = \left[M_p - (M_p - M_r) \left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p} \right) \right] \quad (83)$$

with $M_r = F_y S$ (84)

S = the elastic section modulus in LRFD

iii. For $\lambda \geq \lambda_r$,

$$M_n = F_y S_{eff} \quad (85)$$

where

S_{eff} = the effective section modulus with the effective width of the compression flange given as in Eq. (90)

In addition, the nominal critical stress F_{cr} is evaluated as follows:

i. For $\lambda_c \sqrt{Q} \leq 1.5$,

$$F_{cr} = Q (0.658^{Q\lambda_c^2}) F_y \quad (86)$$

ii. For $\lambda_c \sqrt{Q} > 1.5$,

$$F_{cr} = \left(\frac{0.877}{\lambda_c^2} \right) F_y \quad (87)$$

where

λ_c = the slenderness parameter

$$= \frac{kl_e}{\pi r} \sqrt{\frac{F_y}{E}} \quad (88)$$

F_y = the yield stress of steel in ksi (N/mm²)

l_e = the effective length of member

k = the effective length factor = 1

r = the radius of gyration about the axis of bending

Q = 1.0 (For elements meeting the width-to-thickness ratios, λ_r)
 = $Q_s Q_a$ (For elements not meeting the width-to-thickness ratios, λ_r)

Q_s = 1 (For cross-sections with stiffened elements)

$$Q_a = \frac{A_e}{A_g} \quad (89)$$

where the effective area (A_e) is calculated based on the sum of the effective area of the sides using following equation:

$$\text{when } \frac{b}{t} \geq 1.40 \sqrt{\frac{E}{F_y}},$$

$$b_e = 1.91t \sqrt{\frac{E}{f}} \left[1 - \frac{0.38}{(b/t)} \sqrt{\frac{E}{f}} \right] \quad (90)$$

with $f = \frac{P_u}{A_g}$, otherwise $b_e = b$.

Second Order Effects

The second-order analysis procedure for determining the required strength in column members is given by:

$$M_u = B_1 M_{nt} \quad (91)$$

M_{nt} is required flexural strength in member assuming there is no lateral translation. The moment amplifier (B_1) is used to account second order effects and is expressed as,

$$B_1 = \frac{C_m}{\left(1 - \frac{P_u}{P_{el}^2} \right)} \quad (92)$$

where

$$\begin{aligned} C_m &= \text{the coefficient assuming no lateral translation of the member} \\ &= 0.6 - 0.4(M_1/M_2) \\ &= 1.0 \text{ (Assumption and ideal condition)} \end{aligned} \quad (93)$$

P_u = the required axial compressive strength for the member

$$P_{el} = \text{elastic critical buckling strength of the member in the plane of bending}$$

$$= \frac{\pi^2 EI}{kl^2} \quad (94)$$

E = the modulus elasticity of the steel

I = the moment of inertia in the plane of bending

l = the effective length of the member

k = the effective length factor in the plane of bending
= 1 (Assume no lateral translation)

B_1 is calculated at each value of the applied load P_u . The second order moments may be expressed as $B_1 M_u$.

From Eqs. (80) and (81), the applied moment taking into account second order effects, $B_1 M_u$, can be calculated from $B_1 P_u e$, where P_u and e are the applied load and eccentricity of the axial force applied to the column, respectively. The biaxial load is then calculated by Eqs. (80) and (81), taking into account second order effects.

4.4.3 AISC-LRFD design rules for composite columns

The moment capacity of a concrete-filled tube column can be determined using the interaction equation given in Eq. (95) as suggested by Viest et al. [13]. For symmetric members, the bending strength ratio (M_u/M_n) is evaluated separately for each of the two principal axes. The two components are then added to the axial force component to produce linear superposition of bending strength ratios of a biaxially loaded column.

$$\frac{P_u}{P_n} + \frac{M_{ux}}{M_{nx}} + \frac{M_{uy}}{M_{ny}} \leq 1 \quad (95)$$

where

P_u = the required axial capacity

P_n = the nominal resistance in compression, kips (N) = $A_g F_{cr}$

A_g = the gross cross-sectional area of steel

M_{nx}, M_{ny} = the nominal bending strength with bending only in the plane of x and y - axis
respectively

The nominal moment capacity (M_n) of a column cross-section without axial load may be determined from a plastic stress distribution on the steel alone [11].

$$M_n = Z F_y \quad (96)$$

where

Z = the plastic section modulus of the steel casing

F_y = the plastic yield stress of steel in ksi (N/mm²)

In addition, the nominal critical stress (F_{cr}) is evaluated as follows:

i. For $\lambda_c \leq 1.5$,

$$F_{cr} = \left(0.658^{\lambda_c^2} \right) \quad (97)$$

ii. For $\lambda_c > 1.5$,

$$F_{cr} = \left(\frac{0.877}{\lambda_c^2} \right) F_y \quad (98)$$

where

λ_c = the slenderness parameter

$$= \frac{kl_e}{r\pi} \sqrt{\frac{F_{my}}{E_m}} \quad (99)$$

F_{my} = the modified yield stress of steel in ksi (N/mm²)

$$= F_y + c_2 f_c' \left(\frac{A_c}{A_s} \right) \quad (100)$$

A_s = the cross-sectional area of steel

A_c = the cross-sectional area of concrete

F_y = the yield stress of steel

c_2 = 0.85

f_c' = the specified compressive strength of concrete

l_e = the effective length of the member

k = the effective length factor
= 1

r = the radius of gyration about the axis of bending

The modified modulus elasticity (E_m) is expressed as,

$$E_m = E_s + c_3 E_c \left(\frac{A_c}{A_s} \right) \quad (101)$$

c_3 = 0.4

A_s = the cross-sectional area of steel

A_c = the cross-sectional area of concrete

E_s = the modulus elasticity of the steel

E_c = the modulus elasticity of the concrete

Second Order Effects

The second-order analysis procedure specified in LRFD for determining the required strength in composite column members is similar to the approach used in hollow steel members given as follows,

$$M_u = B_1 M_{nt} \quad (102)$$

M_{nt} is required flexural strength in member assuming there is no lateral translation. The moment amplifier B_1 is used to account for second order effects and is expressed as,

$$B_1 = \frac{C_m}{\left(1 - \frac{P_u}{P_{el}}\right)} \quad (103)$$

where

$$\begin{aligned} C_m &= \text{the coefficient assuming no lateral translation of the member} \\ &= 0.6 - 0.4 (M_1/M_2) \\ &= 1.0 \text{ (Assumption of an ideal condition)} \end{aligned} \quad (104)$$

P_u = the required axial compressive strength for the member

$$\begin{aligned} P_{el} &= \text{elastic critical buckling strength of the member in the plane of bending} \\ &= \frac{\pi^2 (E_s I_s + 0.4 E_c I_c (A_c/A_s))}{kl^2} \end{aligned} \quad (105)$$

E = the modulus elasticity of the steel

I = the moment of inertia in the plane of bending

l = the effective length of the member

k = the effective length factor in the plane of bending
= 1 (Assume no lateral translation)

B_1 is calculated at each value of the applied load (P_u). The second order moments may be expressed as $B_1 M_u$.

From Eq. (95), the applied moment taking into account second order effects, $B_1 M_u$, can be calculated from $B_1 P_u e$, where P_u and e are the applied load and eccentricity of the axial force applied to the column, respectively. The biaxial load is then calculated by using Eq. (95), taking into account second order effects.

5. COMPARISON WITH INTERNATIONAL CODES OF PRACTICE

The experiments conducted in part I of this paper are quite novel and help to elucidate the behaviour of both hollow and concrete filled high strength steel sections subjected to biaxial loading conditions. However, for design purposes, it is extremely important to have a routine design procedure which is representative, consistent and conservative in the prediction of the ultimate strength. The test results have each been compared against the design models proposed by the AS4100 (Standards Australia, 1998), Eurocode 3 and 4, (British Standards Institution, 1992 and 1994) and LRFD (American Institute of Steel Construction, 2000).

Comparisons are made in Table 1 for the short hollow steel columns. For all the cross-sections the test results are shown to be on the conservative side, except for those proposed by the LRFD. The mean values of the test results to predicted results were 1.39, 1.05 and 0.98 for the AS4100, EC3 and LRFD approaches respectively.

Table 1. Comparisons of Test Results of Short Hollow Steel Columns with International Code Provisions

Column no.	N_{test} (kN)	EC 3 (kN)	AS 4100 (kN)	LRFD (kN)	$\frac{N_{test}}{N_{pred,EC3}}$	$\frac{N_{test}}{N_{pred,AS4100}}$	$\frac{N_{test}}{N_{pred,LRFD}}$
SH-H110	1395	1163	1062	1081	1.20	1.31	1.29
SH-H160	-	1316	963	1441	-	-	-
SH-H210	1421	1444	1017	1659	0.98	1.40	0.86
SH-H260	1471	1536	1010	1836	0.96	1.46	0.80
Mean					1.05	1.39	0.98
Standard deviation					0.13	0.07	0.33

Table 2 gives a comparison of test results for the short concrete filled steel columns against the AS4100, EC4 and LRFD approaches. It is firstly worth highlighting that in order to obtain the AS4100 results the concrete was transformed to an equivalent area of steel. The mean values of the test results to predicted results were 1.09, 1.12 and 1.24 for the AS4100, EC4 and LRFD approaches respectively.

Table 2. Comparisons of Test Results of Short Composite Columns with International Code Provisions

Column no.	N_{test} (kN)	EC4 (kN)	AS 4100 (Concrete- to-Steel) (kN)	LRFD (kN)	$\frac{N_{test}}{N_{pred,EC4}}$	$\frac{N_{test}}{N_{pred,AS4100(c-t-s)}}$	$\frac{N_{test}}{N_{pred,LRFD}}$
SH – C110	1609	1403	1610	1289	1.15	1.00	1.25
SH – C160	2362	2154	2016	1926	1.10	1.17	1.23
SH – C210	-	2973	2468	2591	-	-	-
SH – C260	-	3873	2918	3283	-	-	-
Mean					1.12	1.09	1.24
Standard deviation					0.04	0.12	0.02

For slender columns, a comparison has been made between the axial loads applied in the experiments and those obtained by using the various design codes, which is reflected in Tables 3

and 4. For the hollow steel sections, comparisons with Eurocode 3 (British Standards Institution, 1992), AS 4100 (Standards Australia, 1998) and LRFD (American Institute of Steel Construction, 1993) were made. For the concrete filled steel columns, Eurocode 4 (British Standards Institution, 1994), the suggested approach for AS 4100 (Standards Australia, 1998) and the LRFD approach were used for comparison. The results in Tables 3 shows that both the Eurocodes (EC3) and AS4100 ($\alpha_b=0$) and ($\alpha_b=-0.5$) models are conservative in their predictions with mean values of test strengths to predictions of 1.25, 1.41 and 1.36 respectively. Likewise, the mean values of the test results to predicted results for LRFD approach was 0.92. Table 4, shows that the EC4, AS4100 and LRFD models gives conservative predictions with mean values of test strengths to predictions more than unity. Figure 16 gives a comparison of AS4100 strength curves with test results of slender hollow columns with varying eccentricities. This shows the conservative nature of the AS4100 code specification in predicting the strength of the slender columns under biaxial bending.

Table 3. Comparisons of Test Results Slender Hollow Steel Columns with International Code Provisions

Column no.	N_{test} (kN)	EC 3 (kN)	AS4100 ($\alpha_b=0$) (kN)	AS4100 ($\alpha_b=-0.5$) (kN)	LRFD (kN)	$\frac{N_{test}}{N_{pred,EC3}}$	$\frac{N_{test}}{N_{pred,AS4100}}$ ($\alpha_b=0$)	$\frac{N_{test}}{N_{pred,AS4100}}$ ($\alpha_b=-0.5$)	$\frac{N_{test}}{N_{pred,LRFD}}$
SL-H110	642	539	515	542	677	1.19	1.25	1.18	0.95
SL-H160	1230	861	809	844	1031	1.43	1.52	1.46	1.19
SL-H210	1300	1076	928	955	1535	1.21	1.40	1.36	0.85
SL40-H210	1077	814	716	732	1260	1.32	1.50	1.47	0.85
SL-H260	1342	1215	980	1000	1760	1.10	1.37	1.34	0.76
Mean						1.25	1.41	1.36	0.92
Standard deviation						0.13	0.11	0.12	0.17

Figure 16. Comparison of AS4100 Strength Curves with Test Results of Slender Hollow Columns with Varying Eccentricities

The experiments presented herein have been compared with the currently available design methods for columns under biaxial loading as outlined in the existing European, Australian and American Standards. The results have shown that the standards are quite conservative in their prediction of the ultimate loads. The models are therefore quite adequate for design, but there may be some scope for improvements of the method to allow optimisation of the columns when used for this application.

Table 4. Comparisons of Test Results of Slender Composite Columns with International Code Provisions

Column no.	N_{test} (kN)	EC 4 (kN)	AS4100 (c-t-s) ($\alpha_b=0$) (kN)	AS4100 (c-t-s) ($\alpha_b=-0.5$) (kN)	LRFD (kN)	$\frac{N_{test}}{N_{pred,EC4}}$	$\frac{N_{test}}{N_{pred,AS4100}}$ ($\alpha_b=0$)	$\frac{N_{test}}{N_{pred,AS4100}}$ ($\alpha_b=-0.5$)	$\frac{N_{test}}{N_{pred,LRFD}}$
SL-C110	765	656	665	696	741	1.17	1.15	1.10	1.03
SL-C160	1858	1469	1370	1433	1522	1.26	1.36	1.30	1.22
SL-C210	2784	2333	2035	2102	2277	1.19	1.37	1.32	1.22
SL40-C210	2188	1824	1425	1442	1693	1.20	1.54	1.52	1.29
SL-C260	3634	3289	2674	2740	2932	1.10	1.36	1.33	1.24
Mean						1.19	1.35	1.31	1.20
Standard deviation						0.06	0.14	0.15	0.10

Note: *c-t-s* – refers to the method of transformation of concrete to an equivalent of steel

6. CONCLUSIONS

This paper has presented a numerical model aimed at defining the point of local buckling, and predicting the ultimate strengths of the sections under orthogonal loading incorporating local and post local buckling. The model has been calibrated with experiments conducted by the authors, and have been compared with existing design code specifications in AS4100, Eurocode 3, Eurocode 4 and LRFD to consider its applicability for design.

The numerical model was shown to be satisfactory in defining the points where local buckling takes place in the component steel plates. This model also addresses the interaction buckling modes considering coupled effects of confinement and local buckling before and after the peak range for composite columns. The results produced by the model show the load moment response of the columns, and good agreement was indicated with the experimental results. Local buckling effects play a major role in considering the confinement effects provided by the concrete core and is governed by the slenderness of the component plates of the column. The model and the experiments have considered a wide range of plate slenderness in which elastic and inelastic local buckling occurs. In slender cross-sections, the load moment curve exceeds the unconfined curve, hence, this shows there is a reserve strength in the columns.

Good agreement was attained between the ultimate strengths from tests and the values predicted by international code provisions. The accuracy of existing code specifications from various design codes has been substantiated through comparisons with experiments of hollow and composite columns subjected to biaxial loading. The AS4100 design code was found to be the most conservative in predicting the strength of hollow short and slender high strength steel columns under biaxial bending. Compared with the analytical study, the predictions using the EC3 and the AS4100 specifications are fairly conservative for the hollow specimens. In comparison with the ultimate strength of concrete-filled steel columns, the strength of hollow steel columns are much lower than the concrete-filled columns. This is due to the contribution of concrete in the steel

section, which delays local buckling in the steel casing. Hence, the methods discussed herein are considered appropriate for conservatively predicting the actual failure strength of hollow and concrete-filled high strength steel columns under biaxial bending.

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