MINLP OPTIMIZATION OF STEEL FRAMES

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ABSTRACT: The paper presents the discrete dimension optimization of unbraced rigid steel plane frames. The optimization of steel frames was carried out by the Mixed-Integer Non-linear Programming (MINLP) approach. The MINLP is a combined discrete-continuous optimization technique. It performs the discrete optimization of discrete decisions simultaneously with the continuous optimization of continuous parameters. The task of the optimization is to minimize the mass of the frame structure and to find the optimal discrete sizes of standard steel sections for frame members. The finite element equations are defined as the equality constraints for the second-order elastic structural analysis. The design constraints for the steel members were formulated according to Eurocode 3. The Modified Outer-Approximation/ Equality-Relaxation algorithm and a two-phase MINLP optimization approach were applied for the optimization. The latter starts with the continuous optimization of the frame, while the standard dimensions are temporarily relaxed into continuous parameters. When the optimal continuous solution is found, standard sizes of cross-sections are re-established and the simultaneous continuous and discrete dimension optimization by MINLP is then continued until the optimal solution is found. A numerical example of the optimization of a steel frame is presented at the end of the paper to show the suitability of the proposed approach.

Keywords: Optimization; mixed-integer non-linear programming; MINLP; steel structures; frames; Eurocode 3

1. INTRODUCTION

Traditional engineering methods for the structural design of steel frames are based on a trial-and-error procedure. The effective design is achieved in a time-consuming procedure of analyzing different structural alternatives by varying the sizes of the steel members. However, doubt always exists as to whether or not the obtained structural design is optimal.

To surmount the mentioned disadvantages, various different techniques, suited either for the continuous or the discrete structural optimization, have been proposed over the last three decades. These modern optimization methods may be partitioned into mathematical programming methods and heuristic methods.

As regards the mathematical programming, the non-convex and non-linear optimization problem of steel frames has in most cases been solved by using different continuous Nonlinear Programming (NLP) methods based on the well known Karush-Kuhn-Tucker optimality conditions, Kuhn HW and Tucker [1]. In this field, the Optimality Criteria methods (OC) are one of the most frequently used approaches. The OC methods were developed on the basis of the contributions by several researchers in the 1960s and early 1970s such as Barnett [2], Prager and Shield [3] and Venkayya et al. [4]. In the recent years, the OC approach to the optimization of steel frames was proposed by Chan et al. [5], Soegiarso and Adeli [6], Saka and Kameshki [7]. While the Karush-Kuhn-Tucker conditions ensure the requirements for the optimal solution, the Lagrange multipliers are applied to comprise the constraints. The OC methods handle continuous design variables. In cases when discrete variables are required, an optimum solution is commonly obtained in two steps. In the first step, the optimization problem is solved using the continuous variables. In the second step the discrete values are estimated by matching or rounding the values obtained from the continuous solution.

Since the nature of the optimization problem of steel frames is discrete, meaning that the structural design of steel frame members is determined by the discrete standard sizes, the nonlinear discrete optimization must be applied to handle discrete variables explicitly. In this way, various heuristic methods have been developed and mainly used: e.g. Direct Search (DS), Hook and Jeeves [8], Genetic Algorithms (GA), Holland [9], Simulated Annealing (SA), Kirkpatrick et al. [10], Tabu Search (TS), Glover [11], Neural Networks (NN), Rumelhart et al. [12], Ant Colony Optimization (ACO), Dorigo et al. [13], etc. In the class of heuristic methods, the GA are probably the most frequently applied approaches. GA are search algorithms based on the principles of natural selection and mutation. Not to go into too many details concerning the GA, some of their basic characteristic should be described: GA work with an encoded set of variables and operate on a population of potential solutions. They use a transition scheme that is probabilistic and an objective function information without any gradient information, and are able to deal with discrete optimum design problems without the derivates of functions. Only few of the numerous contributions of the frame optimization based on the GA, are brought to attention here: Camp et al. [14], Erbatur et al. [15], Kameshki and Saka [16], Jármai et al. [17], Hayalioglu and Degertekin [18], Kim et al. [19], Kaveh and Abdietehrani [20], Sarma and Adeli [21].

In recent years, some research contributions were presented on the optimization of steel frames with the Eurocode 3 design constraints for the dimensioning. In most of the published research works, different optimization methods were combined with the elastic first-order structural analysis. Guerlement et al. [22] have presented a practical sequential optimization algorithm which was used for discrete optimization of steel portal frames. Uys et al. [23] have used the leap-frog gradient method for the optimal design of the steel hoist structure frame. A gradient based optimization algorithm was used to obtain a continuous solution, which was then utilized as the starting point for a neighbourhood search within the discrete set of profiles available to attain the discrete optimal design. Jármai et al. [17] have investigated the suitability of four different optimization algorithms (i.e. genetic algorithm, the leap-frog gradient method, the method of Rosenbrock and the differential evolution algorithm) for the optimal design of the welded steel I-section frame. Further, Jármai et al. [24] have presented the optimum seismic design of a three-storey steel frame, obtained by the particle swarm optimization algorithm. Krajnc and Beg [25] have performed the weight optimization of welded frame structures, where the optimization was carried out by the genetic algorithm combined with elastic second-order structural analysis. The elastic second-order structural analysis was based on the approximate stiffness equation, which is the most commonly used method in the engineering practice. The research was then extended to cost optimization of welded steel frames by Pavlovčič et al. [26].

On the other hand, this paper presents the discrete dimension optimization of unbraced rigid steel plane frames performed by the Mixed-Integer Non-linear Programming (MINLP) approach. The MINLP is a type of mathematical programming method which handles both the continuous and the discrete variables simultaneously. Since the MINLP denotes combined discrete-continuous optimization techniques, it performs the discrete optimization of discrete dimensions (i.e. standard cross-section sizes of columns and beams) simultaneously with the continuous optimization of continuous parameters (e.g. internal forces, deflections, the structure mass).

The methodology and the applicability of the proposed MINLP approach to structural optimization problems may be found in the following contributions. Kravanja et al. [27] have presented a general view of the MINLP approach to simultaneous topology and continuous parameter optimization. The development of suitable strategies to solve comprehensive, non-convex and highly combinatorial MINLP problems for the simultaneous topology, material, standard and rounded dimension optimization was introduced by Kravanja et al. [28, 29]. The applicability of the proposed MINLP optimization approach was supported by the examples such as the MINLP optimization of roller hydraulic steel gate Intake Gate of Aswan II for Egypt by Kravanja et al. [30], the optimization of steel gates for Sultartangi project in Iceland [31] as well as the MINLP optimization of steel trusses and composite I beams by Kravanja et al. [29].

The MINLP discrete-continuous optimization problems of steel frames are comprehensive, non-convex and highly non-linear. The Modified Outer-Approximation/Equality-Relaxation algorithm is applied to carry out the structural optimization, Kravanja and Grossmann [32], Kravanja et al. [28, 29]. A two-phase MINLP optimization approach is proposed. It starts with the continuous Non-linear Programming (NLP) optimization of the frame structure, while the standard dimensions are temporarily relaxed into continuous parameters. When the continuous NLP result is found, the standard sizes of the cross-sections are re-established and the simultaneous continuous and discrete dimension optimization by MINLP then continues until the optimal solution is found.

The mass objective function of the frame structure is subjected to the design, material, resistance and deflection constraints taken from the structural analysis. The finite element equations are defined as the equality constraints for the calculation of the internal forces and the deflections of the structure. The second-order elastic structural analysis is performed by considering the geometric nonlinearity due to the P-δ and the P-Δ effect. Both effects are included in the non-linear stiffness matrix of the individual frame member using the stability function approach, Chen and Lui [33]. Design constraints of the steel members are formulated according to Eurocode 3 [34]. Alongside the theoretical basis, a numerical example of the optimization of a steel frame is presented at the end of the paper in order to show the suitability of the proposed approach.

2. STEEL FRAMES

The considered steel frames are proposed to be built as unbraced rigid plane frames from standard hot rolled steel sections, see Figure 1. The discrete dimension optimization of the steel frames is performed under the combined effects of the self-weight of the frame members, the horizontal concentrated variable loads at the left exterior joints, the vertical uniformly distributed variable load at each storey and an initial frame imperfection.

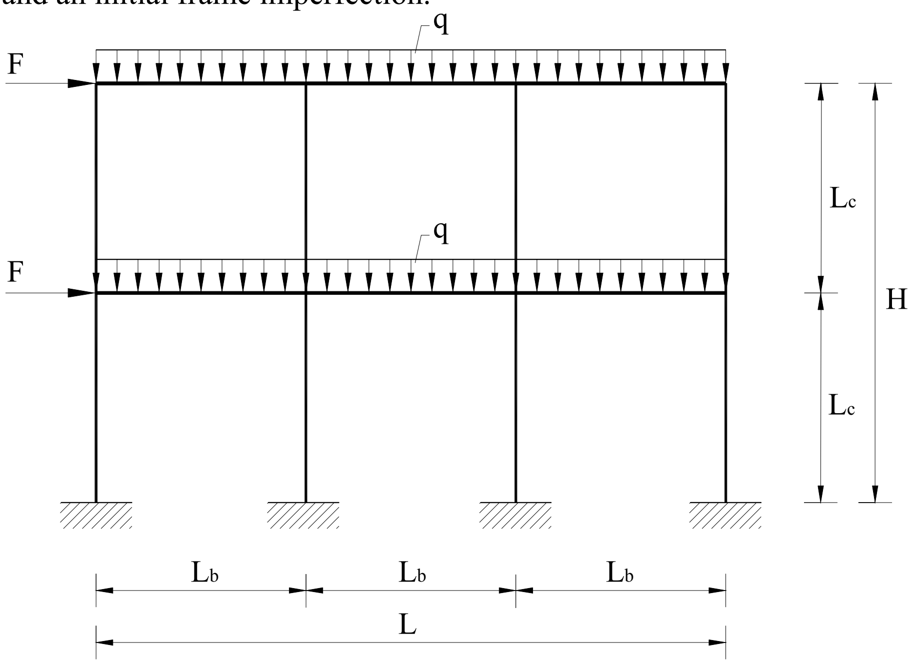

Figure 1. Unbraced Plane Steel Frame

The finite element equations are defined as the equality constraints for the second-order elastic structural analysis. Both the P-δ and the P-Δ effect are included in the non-linear stiffness matrix of the individual frame member using the stability function approach, Chen and Lui [33]. The stability function approach to elastic second-order analysis of steel frames was presented in details by Chen and Lui [35], Chen et al. [36]. While the P-δ effect is associated with the influence of the axial force on the beam-column member flexure, the P-Δ effect denotes the influence of the axial force acting through the relative side sway displacements of the element ends. The stability functions s_{ii} and s_{ij} proposed by Chen WF and Lui [33] are different for tensile and compressive axial forces. Moreover, they give an indeterminate numerical solution when the axial force is zero. To circumvent these problems, Kim et al. [37] have used simplified expressions for the stability functions S_1 and S_2. These expressions are involved in the optimization. Note that the shear deformation effect was neglected considering the fact that only the slender structural members are subjected to buckling for which shear deformation becomes insignificant.

Steel frames are designed in accordance with Eurocode 3 [34] for the conditions of both the ultimate limit and the serviceability limit states. Since the out-of-plane characteristics of the plane structure are unknown, the steel frames are calculated as laterally and torsionally supported frames. Hereby, the steel members are checked only for in-plane instability, i.e. for the compression/buckling resistance of the members.

When the ultimate limit states of the beam-column members are considered, the elements are checked for a required resistance of the cross-section related to local buckling, to the interaction between bending and axial force, to the interaction between bending and axial in-plane compression/buckling, to the effect of both, i.e. the shear and axial forces, on the reduced plastic resistance moment of the cross-section and to shear buckling. The ultimate moment capacity is calculated by the elastic method. Since the second-order elastic global analysis is used, the in-plane buckling lengths of compression members are calculated to be equal to the member's lengths.

When it comes to the serviceability limit states, the vertical deflections of the beams in the individual storey are calculated by the elastic method. The deflections δ_2, resulting from the variable imposed load, and the total deflections δ_{max}, resulting from the overall load, are defined under the limited maximum values: span/300 and span/250, respectively. The horizontal deflections are also checked for each individual storey and for the structure as a whole. Both types of horizontal deflections are checked for the recommended limits: the relative horizontal deflection of each storey required to be smaller than each storey height/300 and the horizontal deflection of the top of the frame must be smaller than the overall frame height/500. Some important ultimate limit state constraints and serviceability limit state constraints are defined in the following sections.

2.1 Ultimate Limit State Constraints

Constraints relating to the steel cross-section requirements for the elastic global analysis and local buckling are given by:

$$d/t_w \leq 42\,\varepsilon\,/\,(0.67 + 0.33\,\psi) \quad \text{for} \quad \psi > -1 \tag{1}$$

$$d/t_w \leq 62\,\varepsilon\,(1-\psi)\,(-\psi)^{0.5} \quad \text{for} \quad \psi \leq -1 \tag{2}$$

$$b/t_f \leq 30\,\varepsilon \tag{3}$$

where:

$$\varepsilon = (235/f_y\ [\text{N/mm}^2])^{0.5}, \qquad \psi = (-M_{y,Sd}/W_{el,y} + N_{x,Sd}/A) \,/\, (M_{y,Sd}/W_{el,y} + N_{x,Sd}/A) \tag{4}$$

where d and t_w are the clear depth and the thickness of the web; b and t_f are the width and the thickness of the flange; f_y is the yield strength of the structural steel; ψ is the stress ratio; $M_{y,Sd}$ is the design bending moment about the y axis; $W_{el,y}$ is the elastic section modulus; $N_{x,Sd}$ is the design axial force and A is the cross-section area.

The constraint concerning the resistance of the steel cross-section subjected to the bending moment and axial force is defined as follows:

$$N_{x,Sd}/(A\, f_y/\gamma_{M0}) + M_{y,Sd}/(W_{el,y}\, f_y/\gamma_{M0}) \le 1 \tag{5}$$

where γ_{M0} is the partial safety coefficient for the plastic, compact and semi-compact sections.

Constraints concerning the resistance of the laterally and torsionally supported beam-column subjected to the bending and axial in-plane compression/buckling may be expressed as follows:

$$N_{x,Sd}/(\chi_y\, A\, f_y/\gamma_{M1}) + k_y\, M_{y,Sd}/(W_{el,y}\, f_y/\gamma_{M1}) \le 1 \tag{6}$$

where:

$$\chi_y = 1/[\Phi_y + (\Phi_y^2 + \overline{\lambda}_y^2)^{0.5}], \qquad \Phi_y = 0.5[1 + \alpha\,(\overline{\lambda}_y - 0.2) + \overline{\lambda}_y^2], \qquad \overline{\lambda}_y = K\,L/[93.9\,\varepsilon\, i_y] \tag{7}$$

$$k_y = 1 - \mu_y\, N_{x,Sd}/(\chi_y\, A\, f_y), \qquad k_y \le 1.5, \qquad \mu_y = \overline{\lambda}_y(2\,\beta_{My} - 4) + (W_{pl,y} - W_{el,y})/W_{el,y}, \qquad \mu_y \le 0.9 \tag{8}$$

where χ_y is the reduction factor for the relevant buckling mode about the y axis; γ_{M1} is the partial coefficient for element instability; α is the imperfection factor; $\overline{\lambda}_y$ is the relative slenderness of the beam-column; K is the buckling length ratio; L is the length of the beam-column; i_y is the radius of gyration; β_{My} is the equivalent uniform moment factor and $W_{pl,y}$ is the plastic section modulus.

Constraints concerning the effect of both shear force and axial force on the reduced plastic resistance moment of the cross-section are written in the following form:

$$V_{z,Sd} \le 0.5\ V_{z,pl,Rd} \tag{9}$$

where:

$$V_{z,pl,Rd} = A_{v,z}\, f_y\, /(\sqrt{3}\ \ \gamma_{M0}), \qquad A_{v,z} = 1.04\ h\ t_w \tag{10}$$

where $V_{z,Sd}$ is the design shear force; $V_{z,pl,Rd}$ is the design plastic shear resistance; $A_{v,z}$ is the steel cross-section shear area and h is the depth of the steel cross-section.

The constraint concerning the resistance to shear buckling is expressed as:

$$d/t_w \le 69\ \varepsilon \tag{11}$$

2.2 Serviceability Limit State Constraints

The constraint concerning the vertical deflections of the beams in each individual storey due to the variable imposed load is determined as:

$$\delta_2 \leq L / 300 \tag{12}$$

where δ_2 is the deflection subjected to the variable imposed load and L is the length of the beam.

The constraint concerning the vertical deflections of the beams in each individual storey due to overall load is written as:

$$\delta_{max} \leq L / 250 \tag{13}$$

where δ_{max} is the total deflection due to overall load.

The constraint concerning the relative horizontal deflection of each storey is stated as:

$$\delta_{hs} \leq h_s / 300 \tag{14}$$

where δ_h is the relative horizontal deflection of the individual storey and h_s is the height of the storey.

The constraint concerning the horizontal deflection of the top of the frame is defined by:

$$\delta_{h0} \leq h_0 / 500 \tag{15}$$

where δ_{h0} is the horizontal deflection of the top of the frame and h_0 is the overall height of the frame.

3. THE MINLP MODEL FORMULATION FOR STEEL FRAMES

The MINLP optimization approach is proposed to be performed through three steps. The first one includes the generation of a mechanical structure with a fixed topology and different standard dimension alternatives, the second one involves the development of an MINLP model formulation and the last one consists of a solution for the defined MINLP optimization problem.

The MINLP structure is proposed to be generated as a mechanical structure consisting of various standard dimension alternatives (e.g. standard hot rolled steel sections) from which an optimal frame structure is obtained within all variations. The selection of the standard dimensions of the alternatives requires a discrete decision optimization.

As soon as the mechanical structure is generated, the MINLP optimization is followed by the development of a MINLP model formulation for steel frames. The proposed MINLP model formulation of frames includes the mass objective function subjected to various constraints with continuous and binary variables. While continuous variables are used for continuous parameter optimization, discrete binary 0-1 variables are used for discrete optimization. They represent the potential selection of standard dimension alternatives which are defined in the mechanical structure.

The non-linear and non-convex discrete/continuous optimization problem of steel frames may be formulated as an MINLP problem in the following form:

$$\begin{aligned}
\min \quad & z = \sum_{i \in I} \rho\, A_i\, L_i \\
\text{s.t.} \quad & \boldsymbol{h}(\boldsymbol{x}) = \mathbf{0} \\
& \boldsymbol{g}(\boldsymbol{x}) \le \mathbf{0} \\
& \boldsymbol{A}(\boldsymbol{x}) \le \boldsymbol{a} \\
& \boldsymbol{S}(\boldsymbol{d}^{st}) \le \boldsymbol{s} \\
& \boldsymbol{x} \in X = \{\boldsymbol{x} \in R^n : \boldsymbol{x}^{\mathrm{LO}} \le \boldsymbol{x} \le \boldsymbol{x}^{\mathrm{UP}}\} \\
& \boldsymbol{y} \in Y = \{0,1\}^m
\end{aligned} \tag{MINLP}$$

where z is an objective function; ρ represents the steel density; A_i is a cross-section area of the i-th structural element; L_i stands for a length of the i-th structural element; I represents a set of structural elements; $\boldsymbol{x}$ is a vector of continuous variables specified in the compact set X; and $\boldsymbol{y}$ denotes a vector of 0-1 discrete binary variables defined inside the set Y. The functions $\boldsymbol{h}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ are non-linear functions involved in the equality and inequality constraints, respectively. $\boldsymbol{A}(\boldsymbol{x}) = \boldsymbol{a}$ represent linear constraints and $\boldsymbol{S}(\boldsymbol{d}^{st}) \le \boldsymbol{s}$ are mixed linear equality/inequality constraints.

Both, the vector of continuous variables $\boldsymbol{x} = \{\boldsymbol{d}, \boldsymbol{p}\}$ and the vector of discrete binary variables $\boldsymbol{y}^{st}$ are involved in the MINLP frame model formulation. The continuous variables are partitioned into design variables $\boldsymbol{d} = \{\boldsymbol{d}^{cn}, \boldsymbol{d}^{st}\}$ and into performance (non-design) variables $\boldsymbol{p}$ (e.g. internal forces, deflections), where sub-vectors $\boldsymbol{d}^{cn}$ and $\boldsymbol{d}^{st}$ stand for continuous and standard dimensions, respectively. The vector of binary variables $\boldsymbol{y}^{st}$ denotes the potential selection of standard dimension alternatives.

The objective function z defines the steel frame structure mass including i, $i \in I$, structural elements (beams and columns). Parameter non-linear and linear constraints $\boldsymbol{h}(\boldsymbol{x}) = \mathbf{0}$, $\boldsymbol{g}(\boldsymbol{x}) \le \mathbf{0}$ and $\boldsymbol{A}(\boldsymbol{x}) \le \boldsymbol{a}$ represent a rigorous system of design, loading, stress, resistance and deflection constraints known from the structural analysis. The finite element equations (second-order elastic structural analysis) are defined in the set of equality constraints for the calculation of internal forces and deflections, while the constraints for the dimensioning are determined in accordance with Eurocode 3 [34].

Mixed linear constraints $\boldsymbol{S}(\boldsymbol{d}^{st}) \le \boldsymbol{s}$ define the standard design variables $\boldsymbol{d}^{st}$. Each standard dimension d^{st} is determined as a scalar product between its vector of standard dimension constants $\boldsymbol{q}$ and its vector of binary variables $\boldsymbol{y}^{st}$. Only one discrete value can be selected for each standard dimension since:

$$d^{st} = \sum_{i \in I} q_i\, y_i^{st} \tag{16}$$

$$\sum_{i \in I} y_i^{st} = 1 \tag{17}$$

Although the variable d^{st} takes a discrete value of standard section, it is from the mathematical point of view defined as the continuous variable, proposed to be calculated between its lower and upper bounds. The continuous variable d^{st} is determined by using of its subjected vector of discrete binary variables $\boldsymbol{y}^{st}$ according to eqs. (16) and (17).

4. OPTIMIZATION

A general MINLP class of optimization problems can in principle be solved by the following algorithms and their extensions: the Nonlinear Branch and Bound (NBB), proposed by Gupta OK and Ravindran [38]; the Sequential Linear Discrete Programming method (SLDP), by Olsen GR and Vanderplaats [39] and Bremicker et al. [40]; the Extended Cutting Plane method (ECP), by Westerlund and Pettersson [41]; Generalized Benders Decomposition (GBD), by Benders [42] and Geoffrion [43]; the Outer-Approximation method (OA), by Duran and Grossmann [44]; the Feasibility Technique (FT) by Mawengkang and Murtagh [45]; and the LP/NLP based Branch and Bound algorithm (LP/NLP BB) by Quesada and Grossmann [46].

The Nonlinear Branch and Bound method is a direct extension of the original Branch and Bound method which was developed to solve mixed-integer linear optimization problems (MILP), see Land and Doig [47], Dakin [48]. Instead of solving LP relaxed problems, it solves NLP relaxed problems at each node. The method, thus, deals with the sequence of generated continuous NLP subproblems which are performed at each node of a tree enumeration where a subset of relaxed 0-1 discrete variables is successively fixed.

The Sequential Linear Discrete Programming (SLDP) method is an extension of the continuous Sequential Linear Programming (SLP). The SLDP method begins with the creation of a mixed-integer linear approximate problem (MILP) from the nonlinear discrete problem (MINLP). Linear programming techniques are then used to solve the approximate problem. A series of approximations and optimizations is carried out, using the Branch and Bound method, until convergence occurs.

The Extended Cutting Plane method (ECP) is suitable for solving large-scale weakly nonlinear MINLP problems (with a high number of linear, but a few or a reasonable number of nonlinear constraints). This method is based on Kelley's Cutting Plane method, Kelley [49]. Instead of solving LP subproblems as the standard method does, it deals with relaxed MILP subproblem optimization at each iteration. The objective function must be defined linearly.

Unlike the SLDP and the ECP method, both the General Benders Decomposition method (GBD) and the Outer-Approximation one (OA) involve solving an alternative sequence of NLP and MILP optimization subproblems. The NLP subproblem corresponds to the optimization of parameters with 0-1 variables, which are temporarily fixed, and yields an upper bound to the MINLP objective function to be minimized. The MILP master problem predicts a lower bound for the MINLP, where new 0-1 variables are identified. The predicted lower bounds increase monotonically as the cycle of major iterations (MILP plus NLP) proceeds. The search is terminated when the predicted lower bound coincides with or exceeds the current best upper bound. The main difference between the GBD method and the OA one lies in the definition of the MILP master problem. In the case of GBD it is given by a dual representation of the continuous space, while in the case of the OA it is given by a primal approximation. Both methods accumulate new constraints as the major iterations proceed. The GBD accumulates one Lagrangian cut in the space of 0-1 variables, while the OA accumulates a set of linear approximations of nonlinear constraints in the space of both the 0-1 variables and the continuous ones. The Outer-Approximation/Equality-Relaxation method (OA/ER) by Kocis and Grossmann [50] was extended from the OA method to provide for explicit handling of the nonlinear equality constraints of MINLP problems.

The main idea of the Feasibility Technique is to round up the relaxed NLP solution into an integer solution with the least local degradation. This is accomplished through the computer code MINOS by Murtagh and Saunders [51] by successively forcing superbasic variables to become nonbasic ones based on the information of the reduced costs.

The LP/NLP based Branch and Bound method (LP/NLP BB) was proposed for solving convex MINLP problems. The method is based on the solution of both LP and NLP subproblems. NLP subproblems are solved at those nodes in which feasible integer solutions are found.

4.1 The Modified OA/ER Algorithm

Since the MINLP discrete/continuous optimization problems of steel frames are in most cases comprehensive, non-convex and highly non-linear, the Outer-Approximation/Equality-Relaxation algorithm (OA/ER) by Kocis and Grossmann [50] was selected to fulfil this optimization task. The OA/ER algorithm consists of an alternative sequence of solving Non-linear Programming (NLP) optimization subproblems and Mixed-Integer Linear Programming (MILP) optimization master problems, see Figure 2.

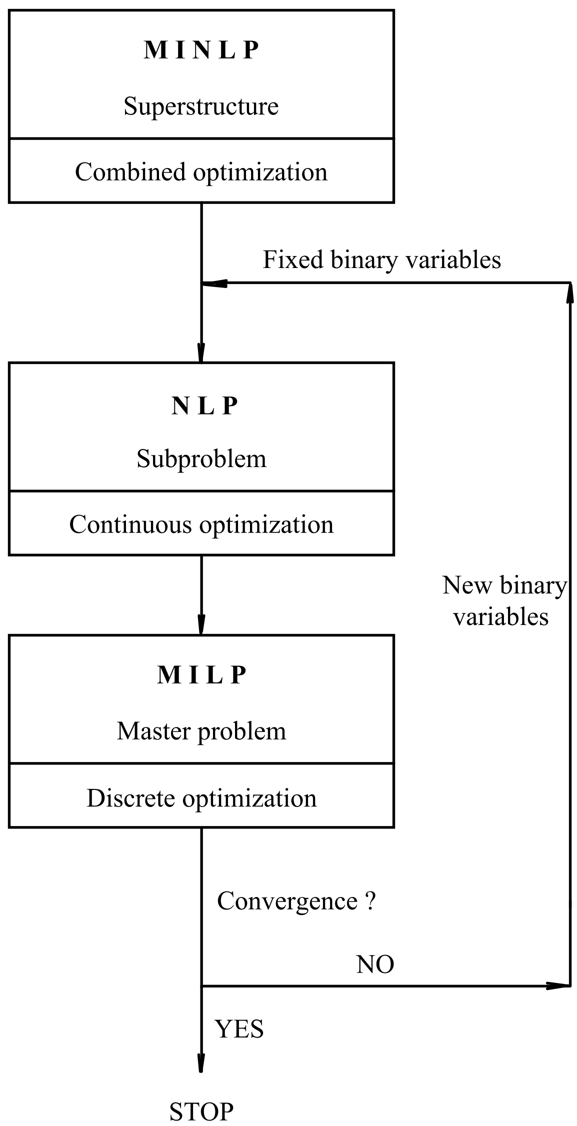

Figure 2. Steps of the OA/ER Algorithm

The NLP optimization subproblem comprises the continuous optimization of the parameters of the frame structure with fixed standard dimensions and yields an upper bound to the objective to be minimized. The MILP optimization master problem predicts a new vector of binary variables. A global linear approximation to the structure of standard dimension alternatives is constructed for the MILP master problem in which new standard sizes (i.e. the standard hot rolled steel sections) are identified in such a way that MILP's lower bound does not exceed the current best NLP's upper bound.

The NLP subproblems and the MILP master problems are sequentially solved until the convergence is satisfied. The search for the optimal solution is terminated when the predicted MILP's lower bound exceeds the current best NLP's upper bound. The convergence is usually achieved in a few MINLP iterations (up to 10). The OA/ER algorithm guarantees the global optimality of solutions for convex and quasi-convex optimization problems.

The OA/ER algorithm as well as all other mentioned MINLP algorithms does not generally guarantee that the obtained solution is the global optimum. This is due to the presence of non-convex functions in the models that may cut off the global optimum. In order to reduce the undesirable effects of non-convexities the Modified OA/ER algorithm was proposed by Kravanja and Grossmann [32] by which the following modifications are applied to the master problem: deactivation of the linearizations, decomposition and deactivation of the objective function linearization, use of the penalty function, use of the upper bound on the objective function to be minimized as well as a global convexity test and a validation of the outer approximations.

Deactivation of the linearizations is a modification procedure proposed by Kocis and Grossmann [52] which establishes the feasibility of the linearizations at zero conditions when the structural element does not exist. An extra existence binary variable y is assigned for each structural element and the set of linear approximations can then be potentially deactivated. The objective here is to enforce linearizations for $y = 1$ (i.e. the structural element is selected), while they become redundant for $y = 0$.

The objective function is usually given in the composite nonlinear form $\boldsymbol{c}^{\mathrm{T}}\boldsymbol{y} + f(\boldsymbol{x})$. Consequently, its linearization also has a composite form, which prevents the above mentioned deactivation procedure from being applied. The objective function is proposed to be decomposed into the mixed linear part $\boldsymbol{c}^{\mathrm{T}}\boldsymbol{y} + f^{\mathrm{lin}}(\boldsymbol{x})$ and into the nonlinear part $f^{\mathrm{nl}}(\boldsymbol{x})$. The nonlinear part $f^{\mathrm{nl}}(\boldsymbol{x})$ of the composite objective function is then further decomposed and represented as a sum of nonlinear terms for each structural element. Since an extra existence variable y is assigned to each nonlinear term, the linearization of each term can be performed separately and the corresponding set of linearizations can potentially be deactivated.

The OA/ER algorithm has been improved by the use of the penalty function which allows violations of the linearizations of nonconvex constraints in the infeasible region despite the nonconvexities and thus makes the obtaining of a feasible solution possible. This can be accomplished by introducing slack variables into any linearization and by including the violations of linearizations with the corresponding weights for slack variables in the penalty function.

Although the shifting of the linearizations into the infeasible region may due to the procedure mentioned above prevent the nonconvexities from cutting off the global optimum, this solution may not be "seen" because of the very high penalties which may be assigned to it. In order to encourage a search into the (in)feasible region, the upper bound UB on the objective function of the MILP master problem is introduced, where the UB is usually set to the currently best NLP solution.

The troublesome impacts of highly nonconvex constraints or terms in the objective function are usually so strong that the original master problem of the OA/ER algorithm fails to predict a good starting point for the next NLP stage. The values of variables are usually shifted either to their upper or to their lower bounds which prevents the next NLP subproblem from converging into its feasible solution. In order to overcome such difficulties, a special global convexity test is applied to the linearizations before each MILP step and then all linearizations that violate the convexity test are made temporarily redundant. Thus, the proposed procedure may accumulate much fewer nonconvexities and prevent violating linearizations from cutting off the feasible region. The convexity conditions must be verified for linearizations for the last point $\boldsymbol{x}^K$. The linearizations which do not satisfy the above conditions are then made currently redundant by simply setting their weights on the slack variables to zero value.

4.2 The Two-phased MINLP Approach

The optimal solution of a complex non-convex and non-linear MINLP problem with a high number of discrete decisions is in general very difficult to obtain. The optimization is thus performed sequentially in two different phases to accelerate the convergence of the Modified OA/ER algorithm. The optimization is proposed to start with the continuous NLP optimization of the frame, while standard dimensions are temporarily relaxed into continuous parameters. When the optimal result is found, the standard sizes of the cross-sections are re-established and the standard dimension optimization of the beams and columns is then continued until the optimal solution is found.

The task of the first phase (the continuous NLP) is to find a good starting point for the MINLP optimization. All linearizations of the nonlinear (in)equality constraints derived at this level, together with the original linear constraints, are valid outer-approximations for the second phase. In this way, each feasible NLP solution of each MINLP iteration (NLP plus MILP) of the second phase accumulates (adds new linearizations) a global linear approximation of the structure model representation to be used at the next MINLP iteration, thus enabling the second phase to be solved much more efficiently, see also Kravanja et al. [27].

The optimization model may contain up to some thousand binary 0-1 variables of standard dimension alternatives. Since this number of 0-1 variables is too high for a normal solution of the MINLP, a special pre-screening and a reduction procedure have been developed, which automatically reduce the binary variables for standard dimension alternatives into a reasonable number. The discrete optimization includes only those 0-1 variables which determine the standard dimension alternatives close to the continuous dimensions obtained at the first NLP.

5. THE EXAMPLE

In order to present the advantages of the proposed MINLP optimization approach, the paper introduces an example of the optimization of a three-storey, three-bay rigid steel plane frame, see Figure 3. The frame was subjected to the combined effects of the self-weight of frame members, the horizontal concentrated variable load of 10 kN at the left exterior joints, the vertical uniformly distributed variable load of 50 kN/m at each storey and the initial frame imperfection. The material used was standard structural steel S 355. The task of the optimization was to minimize the mass of the frame and to find the optimal sizes of standard hot rolled European wide flange HEA sections for the frame members. The optimization of the frame was performed by the MINLP optimization approach.

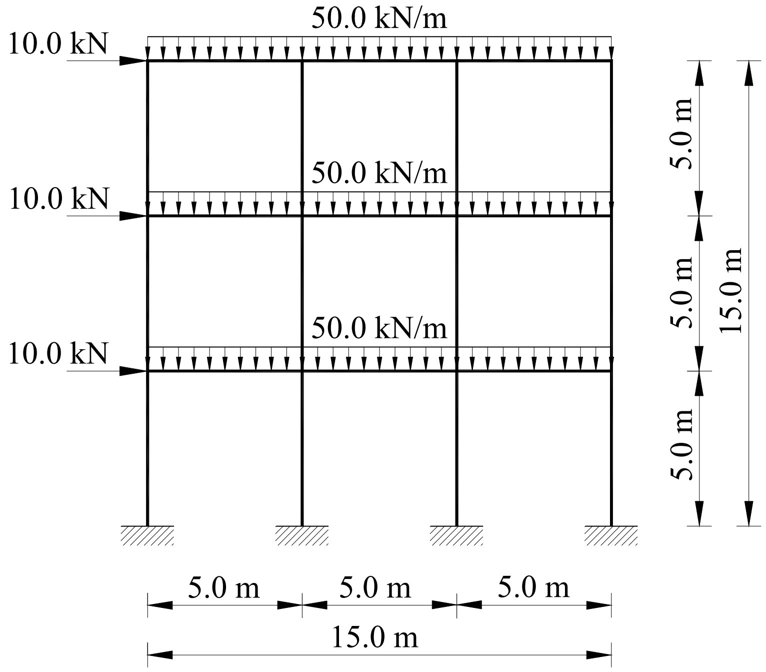

Figure 3. Three-storey, Three-bay Steel Frame

The MINLP optimization model FRAMEOPT for the optimization of steel frames was developed. The model FRAMEOPT was developed on the basis of the proposed MINLP model formulation for steel frames, see paragraph 3. The defined mass objective function of the structure was subjected to the given design, material, resistance, and deflection constraints, checking for both the ultimate and the serviceability limit states according to the Eurocodes, see paragraph 2. As an interface for mathematical modelling and data inputs/outputs GAMS (General Algebraic Modelling System), a high level language was used, see Brooke et al. [53].

A steel frame structure was generated in which all possible variations of different standard sizes were embedded. The structure comprised 17 different standard steel HEA sections (from HEA 100 to HEA 500) for each beam and column separately.

The optimization was carried out by a user-friendly version of the MINLP computer package MIPSYN, the successor of PROSYN Kravanja and Grossmann [32] and TOP Kravanja et al. [54]. The Modified OA/ER algorithm and the two-phased (continuous and then discrete) optimization were applied, where GAMS/CONOPT2 (the Generalized reduced-gradient method) Drudd [55] was used to solve NLP subproblems and GAMS/Cplex 7.0 (Branch and Bound) [56] was used to solve MILP master problems.

The MINLP optimization model of the steel frame contained 1530 (in)equality constraints, 1195 continuous and 357 binary 0-1 variables (61 after the pre-screening). The final optimal solution of 6214 kg was obtained in the 62th main MINLP iteration (the subsequent feasible results were not as good). 30 minutes of overall working time were spent on a 2,13 GHz M and 2 GB of RAM PC to obtain the optimal result. The outline of the optimal frame structure is shown in Figure 4.

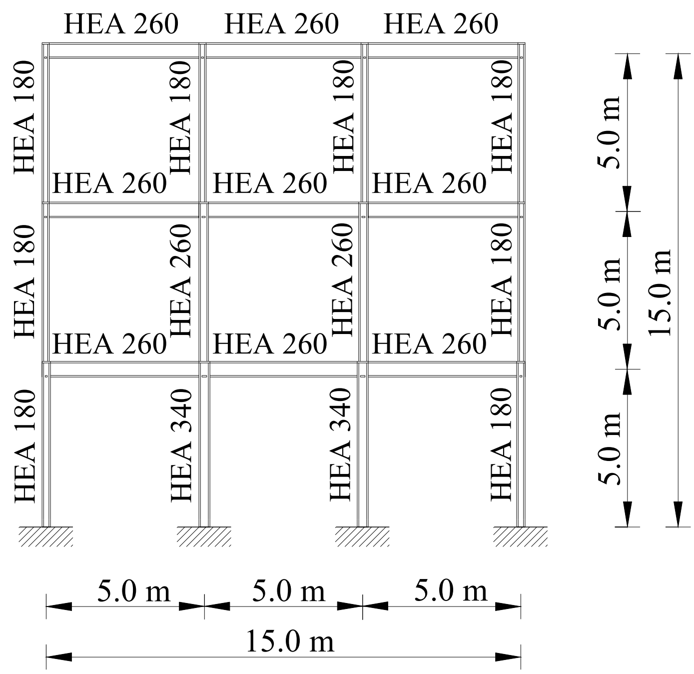

Figure 4. Optimum Design of the Frame

In order to obtain a good starting point for discrete optimization, the optimization of the frame started with the continuous NLP only. All variables and dimensions obtained were the continuous ones. The calculation then proceeded with the discrete optimization of standard sections at the second phase, where 17 different standard section alternatives and thus 17 associated binary 0-1 variables were defined for each structural element separately. In this way, 17×21=357 binary variables were associated with all 21 beams and columns. Since this number of 0-1 variables is high, the pre-screening procedure of alternatives was applied, which automatically reduced the binary variables for standard dimension alternatives into a smaller number. The optimization at the second level included only those 0-1 variables, which determined the standard section alternatives close to the continuous dimensions, obtained at the first NLP. Only 3 binary variables, i.e. 2 variables over and 1 variable under the calculated continuous value, were used for the calculation of each standard section of 19 beams and columns. One variable over and one variable under the continuous value were extra defined for each of two lower inner columns. In this way, only 19×3+2×2=61 binary variables were used at the second level instead of all 357 binary variables, which considerably improved the efficiency of the search.

Alongside the optimal structure mass, the optimal sizes of the standard steel HEA sections of the beams and columns were also obtained. Without the two-phased optimization approach no feasible result was calculated.

6. CONCLUSIONS

The paper presents the Mixed-Integer Non-linear Programming approach (MINLP) to the discrete dimension optimization of unbraced rigid steel plane frames. The optimization is carried out in a single uniform calculating process, where continuous parameters and discrete dimensions are considered simultaneously in order to obtain the minimum mass of a frame structure.

In order to fulfil this task, a frame mechanical structure consisting of various standard dimension cross-section alternatives was generated and the MINLP optimization model FRAMEOPT for steel frames developed. The mass objective function of the frame structure is subjected to a given design, material, resistance, and deflection constraints. The finite element equations (second-order elastic structural analysis) are defined in the set of constraints for the calculation of internal forces and deflections, while the constraints for the dimensioning are determined in accordance with Eurocode 3. The Modified Outer-Approximation/Equality-Relaxation (OA/ER) algorithm is proposed to be used to solve highly combinatorial, non-linear and non-convex optimization problem of frames. The optimization is performed sequentially in two different phases to accelerate the convergence of the Modified OA/ER algorithm and to obtain feasible solutions. In order to achieve a good starting point, the optimization starts with the continuous NLP in the first phase. The MINLP at the re-established discrete dimensions is then performed until the optimal solution is found. Binary variables for standard dimension alternatives are on the basis of the NLP solution pre-screened and reduced into a reasonable number thus enabling a normal solution of the MINLP.

An example of the optimization of a steel frame is presented at the end of the paper in order to interpret the advantages of the proposed MINLP optimization approach. Considering the obtained results, the MINLP was found to be an applicable optimization technique for solving the optimization problems of this type of structures.

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