A NONLINEAR ANALYSIS METHOD OF STEEL FRAMES USING ELEMENT WITH INTERNAL PLASTIC HINGE

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ABSTRACT: A nonlinear analysis method of steel frames using element with internal plastic hinge is proposed. This method can analyze the frame member applied with laterally-distributed loads only using one element even that a plastic hinge appears within the member. By dividing the member into two segments at the location of the maximum moment, the incremental stiffness matrix of the two segments from time t to $t+\mathrm{d}t$ are derived, then the beam element stiffness equation with internal plastic hinge after the static condensation can be obtained. What's more, this method also considers the influences of some geometrical and material nonlinear factors including second-order effect of axial forces, shear deformation, cross-sectional plastification, residual stress and initial imperfection. This method not only overcomes the time-consuming disadvantages of plastic zone method of frame members because of the fine mesh discretization but also makes up for the problem of the traditional plastic hinge element that plastic hinges must form at the elemental ends. Analysis results show that the proposed method is satisfactory.

Keywords: Steel frames; nonlinear analysis; internal plastic hinge; plastic zone method; cross-section plastification

1. INTRODUCTION

The members of a steel frame may be subjected to laterally-distributed loads, so plastic hinges will be formed within the members. The common method [1,2,3,4,5] to treat this case for analysis of the frame is to arrange a node at the location of the plastic hinge within the member to divide the original one element into two or more elements representing the member. This will increase the number of nodes and degrees of freedom for analysis of the frame. Moreover, the traditional element with plastic hinge formed at the end(s) must fix the locations of nodes in advance, which can not suit the case that the locations of the plastic hinge within the member with laterally-distributed loads may vary during the loading process. In this paper, an approach for nonlinear analysis of steel frames using element with internal plastic hinge is proposed. This approach can use one element to simulate one member in a frame even plastic hinge may form within the member.

2. NONLINEAR ANALYSIS METHOD OF STEEL FRAMES

2.1 Plastic Zone Method and Plastic Hinge Method

Over the past 20 years, scholars has developed and validated various methods of performing second-order inelastic analyses on steel frames [1,2,3,4,5]. Most of these studies may be categorized into one of two types: (1) plastic zone method; or (2) plastic hinge method based on the degree of refinement used to represent yielding. The plastic zone method uses the highest refinement while the elasto–plastic hinge method allows for a significant simplification. The load deformation characteristics of the plastic analysis methods are illustrated in Figure 1.

In the plastic zone method[1,2,3,4,5], a frame member is discretized into finite elements, and the cross-section of each finite element is subdivided into many fibers. The deflection at each division along the member is obtained by numerical integration. The incremental load deflection response at each load step, with updated geometry, captures the second-order effects. The residual stress in each fiber is assumed constant since the fibers are sufficiently small. The stress state of each fiber can be explicitly determined, and the gradual spread of yielding traced. A plastic zone analysis eliminates the need for separate member capacity checks since second-order effects, the spread of plasticity, and residual stresses are accounted for directly. As a result, a plastic zone solution is considered ‘exact.’ Although the plastic zone solution may be considered ‘exact,’ it is not applicable to daily use in engineering design, because it is too computationally intensive and too costly. Its applications are limited to: (1) the study of detailed structural behavior; (2) verifying the accuracy of simplified methods; (3) providing comparisons for experimental results; (4) deriving design methods or generating charts for practical use; and (5) application to special design problems.

A more simple and efficient way to represent inelasticity in frames is the elasto–plastic hinge method [1,2,3,4,5]. Here the element remains elastic except at its ends where zero-length plastic hinges form. This method accounts for inelasticity but not the spread of yielding through the section or between the hinges. The effect of residual stresses between hinges is not accounted for either. The elasto–plastic hinge methods may be first- or second-order. In a first-order plastic analysis, nonlinear geometrical effects are considered negligible, and not included in the formulation of the equilibrium equations. As a result, this method predicts the same ultimate load as a conventional rigid plastic analysis would. In a second-order plastic analysis, the effect of the deformed shape is considered. The simplest way to model the geometrical nonlinearities is to use stability functions. These use only one beam-column element to define the second-order effect of an individual member. Stability functions are an efficient and economical method of performing a frame analysis. It has distinct advantage over the plastic zone method for slender members (whose dominant mode of failure is elastic instability) as it compares well with plastic zone solutions. However, for stocky members (which sustain significant yielding), the simple elasto– plastic hinge method over-predicts the capacity of members as it neglects to consider the gradual reduction of stiffness as yielding progresses through and along the member. Consequently, modifications must be made before this method can be proposed for a wide range of framed structures.

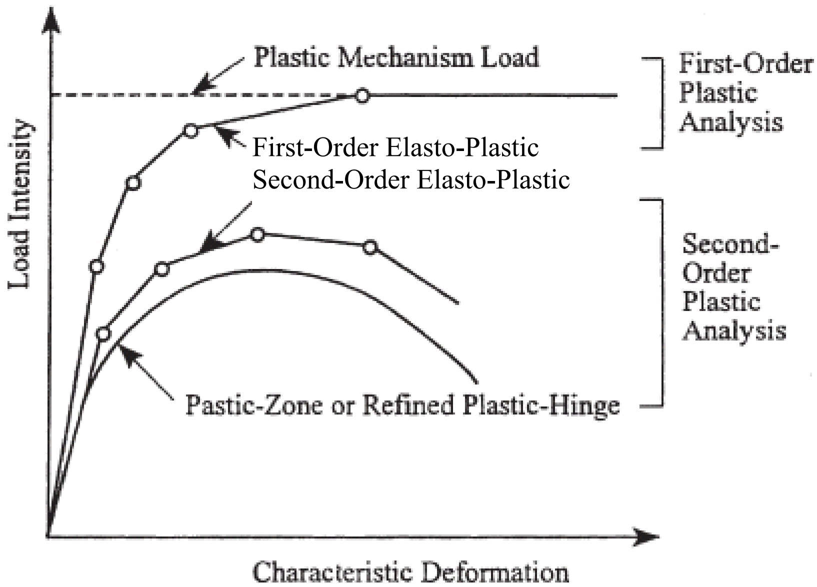


Figure1. Load Deformation Characteristics of Plastic Analysis Methods

2.2 Refined Plastic Hinge Method

A refined plastic hinge analysis incorporates consideration of geometrical and material nonlinear factors including second-order effect of axial forces, shear deformation, cross-sectional plastification, residual stress and initial imperfection to the analysis of steel frames. The concept is outlined in the following section.

2.2.1 Column element

Li and Shen [6] presented an improved plastic hinge model, which considered the cross-section plastification. Using this model the elasto-plastic incremental stiffness equation of the frame column element is given by

$$[k_p]\{\Delta\delta\} = \{\Delta f\} \tag{1}$$

Where, $\{\Delta\delta\}$ and $\{\Delta f\}$ refer to the incremental nodal displacements and forces, respectively, $[k_p]$ refers to the elasto-plastic stiffness matrix, and takes the following form

$$[k_p] = [k_e] - [k_e][G][E][L][E]^T[G]^T[k_e] \tag{2}$$

where

$$[L]^{-1} = [E]^T[G]^T([k_e]+[k_n])[G][E]$$
$$[k_n] = \text{diag}[\alpha_1 k_{e11},\quad \alpha_1 k_{e22},\quad \alpha_1 k_{e33},\quad \alpha_2 k_{e44},\quad \alpha_2 k_{e55},\quad \alpha_2 k_{e66}]$$
$$[E] = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}^T$$
$$[G] = \text{diag}\left[\frac{\partial x_1}{\partial N_1},\quad 0,\quad \frac{\partial x_1}{\partial M_1},\quad \frac{\partial x_2}{\partial N_2},\quad 0,\quad \frac{\partial x_2}{\partial M_2}\right]$$

$[k_e]$ represents the elastic stiffness matrix of the beam element accounting for the second order effect and shearing deformation[6]. In matrix $[G]$, x_i ($i = 1,2$) denotes the ultimate yield surface function of the section. In this paper, the ultimate yield surface function for I type section given by reference [3] and [7] is used here and can be written as

$$x_i = \left(\frac{N}{N_y}\right)^{1.3} + \frac{M}{M_p} = 1 \tag{3}$$

In matrix $[k_n]$, α_i ($i = 1,2$) denotes the elasto-plastic hinge parameter of the two end sections, represents the plastification extent of the two end sections and can be expressed as

$$\alpha_i = \frac{r_i}{1 - r_i} \tag{4}$$

Where, $r_i\,(i=1,2)$ is the restoring force parameter of the two end sections and takes the form as

$$r_i = \begin{cases} 1 & M \le M_{sN} \\ 1 - \dfrac{M - M_{sN}}{M_{pN} - M_{sN}}(1-\beta) & M_{sN} \le M \le M_{pN} \\ \beta & M \ge M_{pN} \end{cases}$$

M , M_{sN} and M_{pN} represent the moment of the section, the initial yield moment the ultimate yield moment under the axial force N , respectively. β represents the material strain hardening coefficient, for normal low carbon steel and low alloyed steel, β can take 0.01~0.02, and M_{pN} can be given by equation (3).

The initial yield surface equation [3,7] without accounting for the influences of residual stress is expressed as

$$\frac{N}{N_y} + \frac{\gamma M}{M_p} = 1.0 \tag{5}$$

and $M_{sN} = (1.0 - \dfrac{N}{N_y})M_p / \gamma$ (6)

The initial yield surface equation [3,7] accounting for the influences of residual stress is expressed as

$$\frac{N}{0.8N_y} + \frac{\gamma M}{0.9M_p} = 1.0 \tag{7}$$

and $M_{pN} = 0.9(1.0 - \dfrac{N}{0.8N_y})M_p / \gamma$ (8)

where, γ is the plastification coefficient of the section.

2.2.2 Beam element with internal plastic hinge

Figure 2 shows the beam element with internal plastic hinge. Referring to Figure 2, an internal node C between elemental ends is inserted so that the element is divided into two segments, the lengths of which are L_a and L_b respectively. Assume the maximum bending moment, M①, at time t is at position C' and the maximum bending moment, M③, at time $t + \mathrm{d}t$ is at position C (see Figure 3). For derivation of incremental stiffness matrix of the element during $t \to t + \mathrm{d}t$, a virtual state of moment, M②, is conceived, which is the bending moment at the same position of M③ at the time t. The incremental stiffness relationship of each segment of the element can be expressed as the standard form as

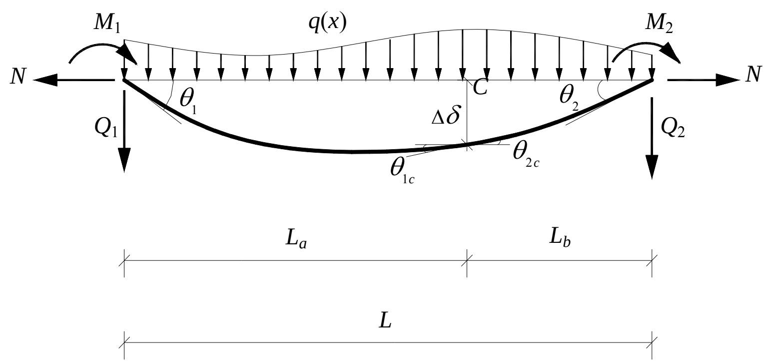


Figure2. Beam Element with Internal Plastic Hinge

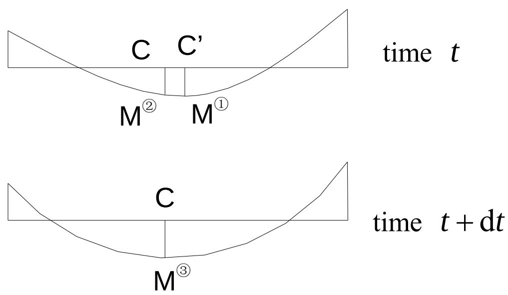


Figure3. Position of Maximum Moment of Different Load Step

for the segment of L_a

$$\begin{Bmatrix} dQ_1 \\ dM_1 \\ dQ_{1c} \\ dM_{1c} \end{Bmatrix} = \left[K_{pa}\right] \begin{Bmatrix} d\delta_1 \\ d\theta_1 \\ d\delta_{1c} \\ d\theta_{1c} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ & a_{22} & a_{23} & a_{24} \\ & & a_{33} & a_{34} \\ & & & a_{44} \end{bmatrix} \begin{Bmatrix} d\delta_1 \\ d\theta_1 \\ d\delta_{1c} \\ d\theta_{1c} \end{Bmatrix} \tag{9a}$$

and for the segment of L_b

$$\begin{Bmatrix} dQ_{2c} \\ dM_{2c} \\ dQ_2 \\ dM_2 \end{Bmatrix} = \left[K_{pb}\right] \begin{Bmatrix} d\delta_{2c} \\ d\theta_{2c} \\ d\delta_2 \\ d\theta_2 \end{Bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ & b_{22} & b_{23} & b_{24} \\ & & b_{33} & b_{34} \\ & & & b_{44} \end{bmatrix} \begin{Bmatrix} d\delta_{2c} \\ d\theta_{2c} \\ d\delta_2 \\ d\theta_2 \end{Bmatrix} \tag{9b}$$

where $\left[K_{pa}\right]$ and $\left[K_{pb}\right]$[6] are the elasto-plastic stiffness matrices for the segments of L_a and L_b of the element respectively, a_{ij} and b_{ij} ($i, j = 1,2,3,4$) are the corresponding elements in such matrices.

It can be seen from Figure 2 that the two segments of the elements share the same deformation components at their junction, namely $d\delta_{1c} = d\delta_{2c} = d\delta_c$ and $d\theta_{1c} = d\theta_{2c} = d\theta_c$. Combining Eq. 9a and Eq. 9b, one has

$$\begin{Bmatrix} \mathrm{d}Q_1 \\ \mathrm{d}M_1 \\ \mathrm{d}Q_2 \\ \mathrm{d}M_2 \\ \mathrm{d}Q_{1c}+\mathrm{d}Q_{2c} \\ \mathrm{d}M_{1c}+\mathrm{d}M_{2c} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & a_{13} & a_{14} \\ & a_{22} & 0 & 0 & a_{23} & a_{24} \\ & & b_{33} & b_{34} & b_{13} & b_{23} \\ & & & b_{44} & b_{14} & b_{24} \\ & & & & a_{33}+b_{11} & a_{34}+b_{12} \\ & & & & & a_{44}+b_{22} \end{bmatrix} \begin{Bmatrix} \mathrm{d}\delta_1 \\ \mathrm{d}\theta_1 \\ \mathrm{d}\delta_2 \\ \mathrm{d}\theta_2 \\ \mathrm{d}\delta_c \\ \mathrm{d}\theta_c \end{Bmatrix} \tag{10}$$

For the purpose of static condensation to eliminate the degree of freedom of the displacements of internal node, above stiffness matrix is partitioned into internal and external degrees of freedom as

$$\begin{Bmatrix} \mathrm{d}f_e \\ \mathrm{d}f_i \end{Bmatrix} = \begin{bmatrix} k_{ee} & k_{ei} \\ k_{ei}^T & k_{ii} \end{bmatrix} \begin{Bmatrix} \mathrm{d}\delta_e \\ \mathrm{d}\delta_i \end{Bmatrix} \tag{11}$$

where $\{\mathrm{d}f_e\}$ and $\{\mathrm{d}f_i\}$ are the elemental end and internal force vectors respectively, $\{\mathrm{d}\delta_e\}$ and $\{\mathrm{d}\delta_i\}$ are elemental end and internal deformation vectors respectively. Their expressions are as follows

$$\{\mathrm{d}f_e\} = [\mathrm{d}Q_1, \ \mathrm{d}M_1, \ \mathrm{d}Q_2, \ \mathrm{d}M_2]^T, \ \{\mathrm{d}\delta_e\} = [\mathrm{d}\delta_1, \ \mathrm{d}\theta_1, \ \mathrm{d}\delta_2, \ \mathrm{d}\theta_2]^T,$$
$$\{\mathrm{d}f_i\} = [\mathrm{d}Q_{1c}+\mathrm{d}Q_{2c}, \ \mathrm{d}M_{1c}+\mathrm{d}M_{2c}]^T, \quad \{\mathrm{d}\delta_i\} = [\mathrm{d}\delta_c, \ \mathrm{d}\theta_c]^T,$$
$$k_{ee} = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{12} & a_{22} & 0 & 0 \\ 0 & 0 & b_{33} & b_{34} \\ 0 & 0 & b_{34} & b_{44} \end{bmatrix}, \quad k_{ei} = \begin{bmatrix} a_{13} & a_{14} \\ a_{23} & a_{24} \\ b_{13} & b_{23} \\ b_{14} & b_{24} \end{bmatrix}, \quad k_{ii} = \begin{bmatrix} a_{33}+b_{11} & a_{34}+b_{12} \\ a_{34}+b_{12} & a_{44}+b_{22} \end{bmatrix} \tag{12}$$

Since no external forces are applied at internal node C, namely $\{\mathrm{d}f_i\} = \{0\}$, $\{\mathrm{d}\delta_i\}$ in Eq. (11) can be expressed with $\{\mathrm{d}\delta_e\}$. The stiffness equation condensed off internal displacement vector is as

$$\left(k_{ee} - k_{ei}k_{ii}^{-1}k_{ei}^T\right)\{\mathrm{d}\delta_e\} = \{\mathrm{d}f_e\} \tag{13}$$

In above derivation, it is assumed that the internal plastic hinge occurs at position of C at time t, and the moment increases from M② at t to M③ at $t+\mathrm{d}t$. But actually in the duration $t \rightarrow t+\mathrm{d}t$, the moment change should have been from M① at position of C' to M③ at position of C. A stiffness matrix modification ($\left[k_{ee} - k_{ei}k_{ii}^{-1}k_{ei}^T\right]_{C,\ t}$ — $\left[k_{ee} - k_{ei}k_{ii}^{-1}k_{ei}^T\right]_{C',\ t}$) may be superimposed to approximately take the effect from position change of internal plastic hinge into account. The subscripts in the stiffness matrix modification indicate the position and the time of maximum bending moment.

Assume the internal plastic hinge occurs at the position of maximum bending moment between two ends. The position of the maximum bending moment between the two ends of the element, position C, varies in loading process. Hence, the rational way to trace the internal plastic hinge is to calculate the position of the maximum bending moment at each loading step after elemental yielding. Two common internal loading patterns for beam elements are concentrated load and uniformly distributed load, as shown in Figure 4.

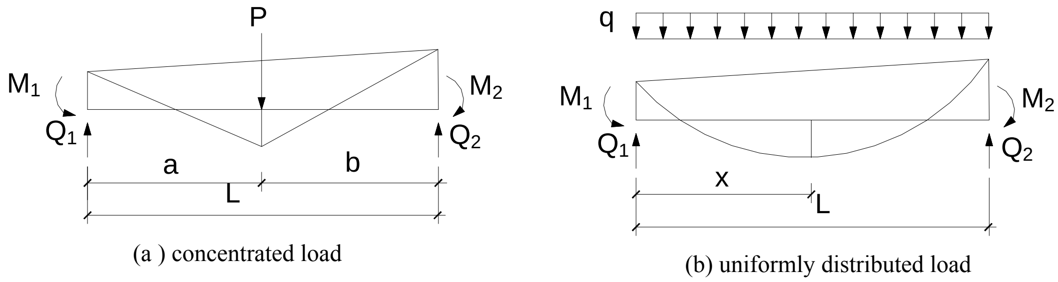


Figure 4. Load Patterns within Beam Span

If one concentrated load is applied within the beam span (see Figure 4a), the position of the maximum moment within span is certainly the loading position. But, if a uniformly distributed load is applied(see Figure 4b), the position of the maximum moment within span is changeable. The condition of the maximum moment within the beam span is that

$$\frac{\mathrm{d}M(x)}{\mathrm{d}x}=0 \quad \text{or} \quad Q(x)=0 \tag{14}$$

The shear force at end 1 can be expressed as

$$Q_1=\frac{M_1-M_2}{L}+\frac{1}{2}qL \tag{15}$$

And letting the shear force be equal to zero yields the position of the maximum moment desired

$$x=\frac{M_1-M_2}{qL}+\frac{1}{2}L \tag{16}$$

As for the beam element with both concentrated load and uniformly distributed load within span, one can divide this element into two segments at the position where the concentrated load applied. The maximum moment position of each segment can be determined according the method for the uniformly distributed load case as above-mentioned. With comparison of the maximum moments of two segments of the element induced by the uniformly distributed load and the bending moment where the concentrated load applied, the real maximum moment of this beam element can be obtained with the maximum of the above three moments.

2.2.3 Residual stress

When the ratio of axial force to squash load is large for a member in compression, residual stresses can influence the plasticity distribution along element length. A transient elastic modulus concept, namely the concept of tangent modulus, is proposed to take this effect into account. The CRC column strength equations [4,5] can be employed in deriving the tangent modulus. The ratio of the tangent modulus to the elastic modulus E_t/E (see Figure 5) is proposed to be

$$\frac{E_t}{E}=1.0 \qquad \text{when} \quad P\le 0.5P_y \tag{17a}$$

$$\frac{E_t}{E}=\frac{4P}{P_y}\left(1-\frac{P}{P_y}\right) \qquad \text{when} \quad P>0.5P_y \tag{17b}$$

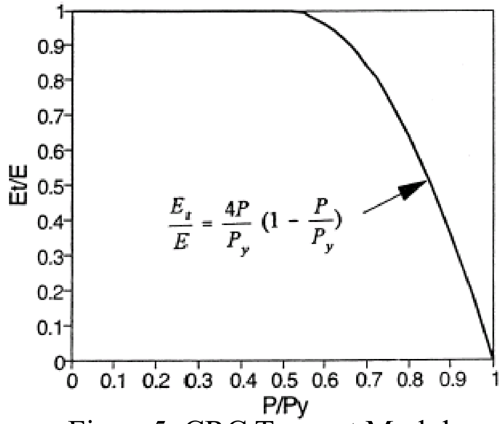


Figure5. CRC Tangent Modulus

2.2.4 Initial imperfections

There are three methods to account for the influences of initial imperfections in the inelastic analysis of steel structures, they are direct modeling, equivalent nominal load method and reduced CRC tangent modulus method [2]. Among all the methods, the third method is the most convenient and direct method which considers the stiffness reduction caused by initial imperfections of the members by multiplying a reduction coefficient. Some research illustrated that the reasonable results can be obtained by taking the reduction coefficient as 0.85. In this paper, the reduced CRC tangent modulus method is used.

3. NUMERICAL EXAMPLES

The structure examined is a four-story frame with mid-span concentrated loads as shown in Figure 6. Table 1 gives the frame member size. The material elastic modulus *E* of steel is 206kN/mm^2, the sections of all the beams are W16×40, the sections of the first storey columns are W12×79 and the sections of the other columns are W10×60.

The horizontal displacement versus load factor curves both obtained by analysis with the elements with internal hinge proposed in this paper and with the normal elements through dividing the frame beam into two elements [8] are shown in Figure 7. The ultimate load factor λ obtained with the proposed elements is 1.03 while that with normal elements [8] is 0.99. The sequence of plastic hinges formed in the frame is illustrated in Figure 8

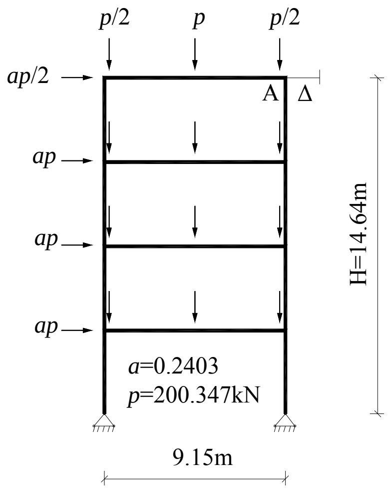


Figure 6. Four-story Frame with Concentrated Loads at Beams

Table 1. Dimensions of All the Components of Four-storey Steel Frame

Section	H(mm)	B(mm)	t_w(mm)	t_f(mm)	A(mm^2)	I(×10^{6}mm^4)
W16×40	406.7	177.5	7.9	12.7	7610	215
W10×60	259.6	256	10.7	17.3	11400	142
W12×79	314.5	306.8	11.9	18.8	15000	276

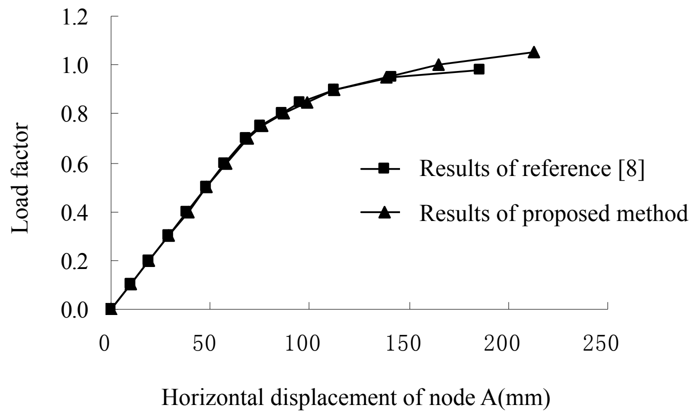


Figure 7. Load-displacement Curve of Four-storey Steel Frame

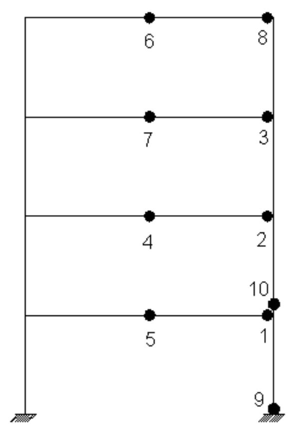


Figure 8. Appearing Sequence of Plastic Hinges of Four-storey Steel Frame

Vogel six-story frame [9] usually appears in benchmark study of planar steel frames. The frame size and load information are illustrated in Figure 9 and the frame member sizes are listed in Table 2. The material elastic modulus E of steel is 206kN/mm^2 and the yield strength f_y is 235N/mm^2. The horizontal displacement of right-upper corner (Node A) versus load factor curve by the elasto-plastic hinge model presented in this paper is compared with the results in reference [9] with plastic zone method in Figure 10. The ultimate load factor λ obtained by the method proposed is 1.15 while that by reference [9] is 1.18. The axial force and moment diagrams in the ultimate state are shown in Figure 11, where the final plastic hinge distribution is dotted in the moment diagram.

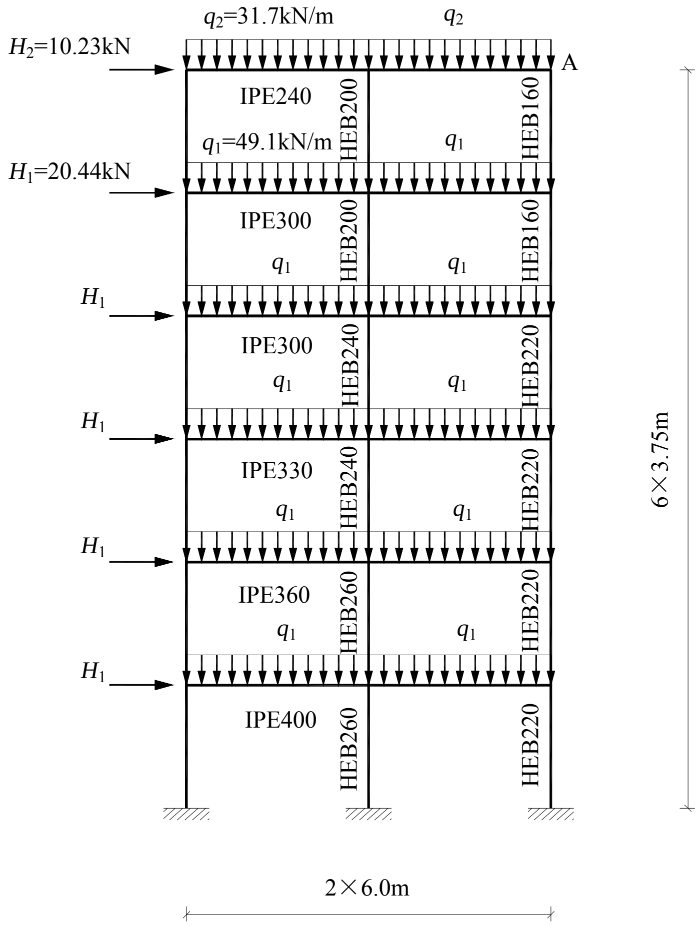


Figure 9. Vogel Six-storey Steel Frame

Table2. Dimensions of all the Components of Vogel Six-storey Steel Frame

Section	H(mm)	B(mm)	t_w(mm)	t_f(mm)	A(mm^2)	I(×10^{6}mm^4)	S(×10^{3}mm^3)
HEA340	330	300	9.5	16.5	13,300	276.9	1850
HEB160	160	160	8.0	13.0	5,430	24.92	354
HEB200	200	200	9.0	15.0	7,810	56.96	643
HEB220	220	220	9.5	16.0	9,100	80.91	827
HEB240	240	240	10.0	17.0	10,600	112.6	1053
HEB260	260	260	10.0	17.5	11,800	149.2	1283
HEB300	300	300	11.0	19.0	14,900	251.7	1869
IPE240	240	120	6.2	9.8	3,910	38.92	367
IPE300	300	150	7.1	10.7	5,380	83.56	628
IPE330	330	160	7.5	11.5	6,260	117.7	804
IPE360	360	170	8.0	12.7	7,270	162.7	1019
IPE400	400	180	8.6	13.5	8,450	231.3	1307

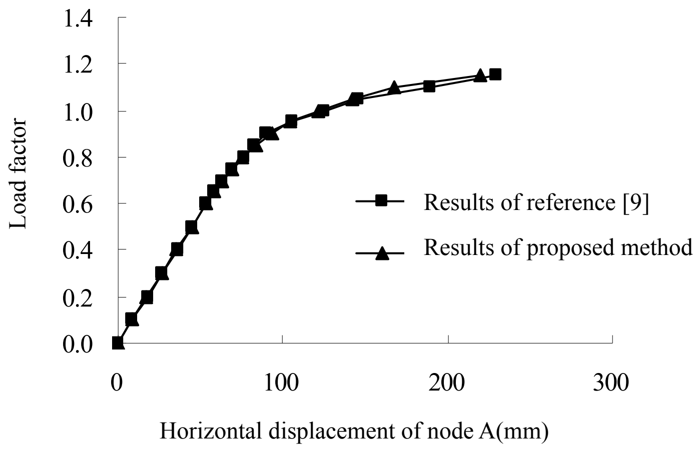
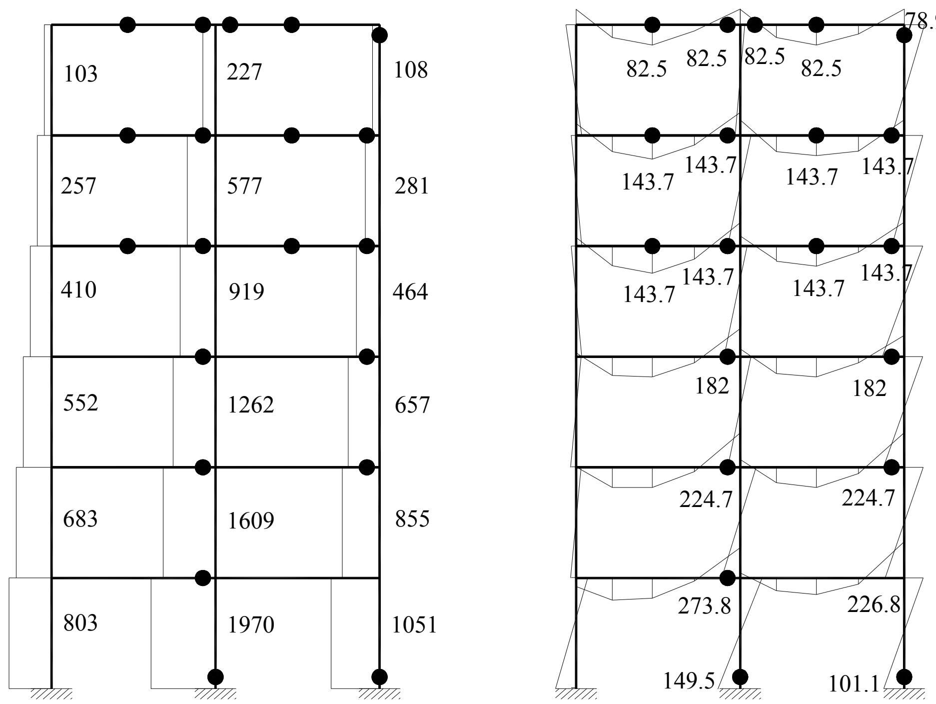


Figure10. Load-displacement Curve of Vogel Six-storey Steel Frame



(a) axial forces (kN) (b) moments (kN·m)

Figure 11. Internal Forces of Vogel six-storey Steel Frame

4. CONCLUSION

An approach for nonlinear analysis of steel frames using element with internal plastic hinge is proposed in this paper. This approach can use just one element to simulate one member in a frame even plastic hinge may form within the member when subjected to laterally-distributed loads. What's more, this approach also considers the influences of some geometrical and material nonlinear factors including second-order effect of axial forces, shear deformation, cross-section plastification, residual stress, initial geometrical imperfection. The numerical results show that the proposed approach is efficient and satisfactorily accurate, and is suitable for the nonlinear analysis of steel frames.

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