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2 ) : ( / ' ( ' & ) & + 6 7 - 2 , 1 7 6

0 L Q \* X / H : H L 7 R , Q J D R / L Q J D Q I G D R X Q ) H Q J = K D Q J

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\$ % 6 7 5 \$ & ) 7 D W L J X H O L I H D V V H V V P H Q W R I Z H Q G V H G F W M R R Q W V & R B G G L R M K F E B E  
I L O O H G & ) F K R U G P H P E H U V X E M V H F M Q L V Q R I R H & L ) & G H I D U R F D K G W L V X Y / L W E D L G C  
L Q I D W L J X H O L I H D V V H V V P H Q W R I H V W L K P D M R H L Q W K H L W W K J R Z / W W B Q D V F Q V I D W  
G H V F U L E H V D P H W K R G R O R J \ I R U F V G B X R O I D W K E J K W W H V S R W H V W J L Q W H L Q V  
7 M R L Q W V E D V H G R Q D I L Q L W H H O B P R Q W H O D Q D \ F I U D F Z N K L F R K G F R Q Q U G D H  
L Q W H U I D F H H O H P H Q W E H F R Q H F L Q H W W H M O S J S I U C R L F H H G X W L P D M R L B Q I R I Z H  
7 M R L Q W V L V D O V R H V W D E O L V K H W R L W F K M V L Q I Q W L D O H F R I D G F N R Q J W K H Q G  
I D W L J X H O L I H D Q G K R W V S R W V W U H W V L R Q O I K I H H P D M I R X I Q G \ W R R F E U H D N V S  
V K D O O R Z F U D F N V W D J H 7 K H S U R S R D H V G R F Q W R K R G W I L L J X H V Q L H B V R I Q D B O  
7 M R L Q W V

. H \ Z R U G R Q F U H W H I L O O H G H F I W L F R X O D : U H O G B O G B V Z R L S Q U H G L F D W L R Q X H ) U D F W X U H  
) L Q L W H H O H P H Q W D Q D O \ V L V

, 1 7 5 2 ' 8 & 7 , 2 1

& R Q F U H W H I L O O H G V W H H O W X E H V Q K B V H Q E J H W V U Z L F G W H X U H X V H G L C  
& K L Q D F R Q F U H W H I L O O H G F L U F X O B Q K Q D O H R Z V L Q B O V L R Q M G & L ) Q  
D U F K W U X V V E U L G J H V 2 Q H H [ D P S G K R L Q J W L K H J : & I L M D % & K L Q D H O R  
D P P D L Q V S D Q , Q W K H & ) & + 6 D U F O R Z U X M F W E R Q G J 8 + 6 F F  
P H P E H U V D Q G W K H E U D F H P H P E H U V D K I R I U Z H V O D W H G I W B O H H V G K Z L W V  
Z K L O H W K H E U D F H V D U H X Q I L O O H G + 6 K / W B R F W X D W / V F C R P R V K V R  
W K D W W K H F R Q F U H W H L Q I L O O F D Q I V X E M W F R Q S U B O D L Y L P S F K R Y H G  
W K H V W L I I Q H V V Q H D U W K H L Q W H B Z H F W U R Q W R I L W K Z H H F O R N Q G R Z Q  
O R D G V P D \ F D X V H I D W L J X H I D L O X U H V D G X H V K M R Z K L Q G W R H H R / V W F  
D Q G Z H O G L Q J G H I H F W V 7 K H Z H O G H G E & ) L G + 6 H M F L L Q H W W K L Q U W I R L M H  
W R I D W L J X H F U D F N L Q J V L Q F H W K H F H O U E H G D E I O H F W U D M R L F \ F O M L  
L P S R U W D Q W L V V X H L Q G H V L J Q L Q J Z H O G H G & ) & + 6 M R L Q W V

( [ W H Q V L Y H U H V H D U F K K D V E H H Q F R O D G X M W I H G W R / Q Z L D W K I R J X M B Q I Z  
H J > @ 7 K H K R W V S R W V W U H G V L R H O H K R L G Q I D Q J E V X H F Q Z H O H G  
M R L Q W V > @ Z K H U H 6 & ) V W U H V V G F R Q F H K Q W U N S R L V R Q W D I F W R U U D  
V W U H Q J W K F X U Y H V D U H U H T X L U H L G O 7 H K H W X I E X V O D Q J M G R L V L W Q B D  
W R Z H O G H G & ) & + 6 M R L Q W V E H F D X V H Y G L M I D U H I Q H M [ S 6 + 6 ] W Y H D G O X 7 K V

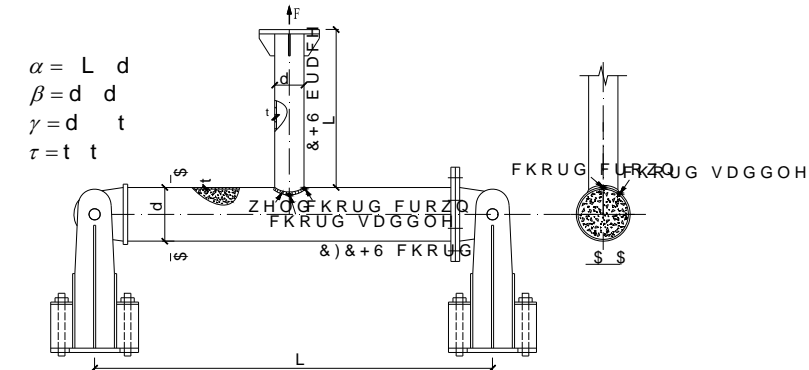
VRPH UHVHDFK RQ & F8+6 D E W H & ) 6 0 0 N 6 F W L X R Q J H M K R R Q W R Z H J  
 . MRLQWV > @ & ) 6 + 6 7 MRLQWV > @ + @ Z B Q I G U & V 8 K + 6 U F N D R U L H C  
 HQRXJK H [SHULPHQWDO GDWD WR H 6 W D E X U V F R W D F R O H W E I O H L O G & H  
 WXEXODU MRLQWV

7KLV SDSHU SUHVHQWV D IUDFWXUHI D W E K D X O L F O V L D H S S R J I R D F I K - M  
 ZKLFK LQYROYHV WKH GHWHUPLQDWLRQ RI R Q V R U H L Q L W Q D O H Q U I D W N I O  
 FUDFN JURZWK PRGHO 7KH VWUHVW KLQ D S S O V L H G M D F W H R U J U V H O  
 DQG JHRPHWU \ > @ \$ EULHI V X P R D U J L R H Q K I Q W I S H B V P R I Q  
 VWUHVHV DQG IDWLJXH OLIH RI Z W O G M G O & ) & R 6 F 7 H M R Q L Q W K V X E  
 DVVXPSWLRQV DERXW WKH FUDFN D X U H O L D I S H D V Q H R V O H Q W W R V E  
 PHFKDQLFV 7R GHWHUPLQH WKH M J X U H G M W L Q M O H Q V L V S U R D F H G K U  
 SDUDPHWHU WUDQVIRUPDWLRQ PHW K R I Q H U H R V L R W U S E B O H E R X G I H  
 ZHOGHG & ) & + 6 7 MRLQW ZLWK VXUI S R I O F W D R Q D D Q G S G I H W S H O D F H  
 SURSDJDWLRQ EHKDYLRU IURP Q X E M U S L F S H O U D Q Q O \ S U M G L F V H G F  
 RI ZHOGHG & ) & + 6 7 MRLQWV DJUHH S H U L O P H Q W D O W K H D W W I R Q E W D L

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7KH GLPHQVLRQV DQG JHRPHWU \ RI Z W O G W I G R & Q & L Q 7 L M R U G W  
 7DEOH \$ WRWDO RI WHQ VSHFLPHQV R Z P H U W H B I Q W F G R Q E K I L F K H  
 RUGHU WR VWXG \ WKHLU H I I H F W M O U L D O M R H G W I R E H K E R Y L R U W I  
 EUDFH LQ WKH MRLQW ZDV VWHHO K H F K R U G E W D X F H R X J D V P D R Q  
 ZHOGLQJ ZLWK IXOO SHQHWUDWLRQ > Z H O G 7 K E D R R R G G Z D D F H L O I  
 FRQFUHHW ZKLOH WKH EUDFH ZDV XQILOOHG

6WDWLF WHVWV LQ WKH HODVWLF H O D G H G & ) H U B 7 L M R W Q F R Q G V  
 8QLYHUVLW \ WR REWDLQ KRW VSRW W W H W H W V F D Q F H Q W R U D R V R I Q  
 GHILQH DV WKH UDWLR RI KRW V Q R W K M W E U D F V D B I G W K K H V H Q E  
 VSHFLPHQV ) R U W K H V W D W L F W D O W W V H Q W K O H E I U R D F H H Z D Z K L V X H M  
 ZDV VLPSO \ V X S S R U W H G V H H ) L J X U H L F D W K E M I S D W L I R H U Q M D F K U  
 PD [ L P X P P 6 ( 8 ) L V W H G L Q 7 D E O H R F F X U U H G D W W K H F K R U G F U R Z



) L J X U H : H O G H G & ) & + 6 7 M R L Q W 6 S H F L P H Q D Q G 6 W D

7DEOH		'HWDLOV RI 7HVW 6SHFLPHQV	
-RLQW	d × t PPiPP	d × t PPiPP	1RQGLPHQVLRQDO JHRPHWULFDO SDUDPHWHU β d d γ dt τ tt
&)&+6	î	î	4 &
&)&+6	î	î	4 &
&)&+6	î	î	4 &
&)&+6	î	î	4 &
&)&+6	î	î	4 &
&)&+6	î	î	4 &
&)&+6	î	î	4 &
&)&+6	î	î	4 &
&)&+6	î	î	4 &
&)&+6	î	î	4 &

1RWH %RWK WJKHVLVH DVG VRIUGHWHJW K D QGH WKH3DLB  
 -RXOHV UHVSHFWLYHO\ 7KH FXELVHF RPSLHV D QGH &VWUHQJWK R  
 DQG 03D UHVSHFWLYHO\

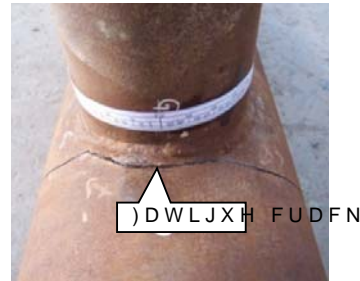
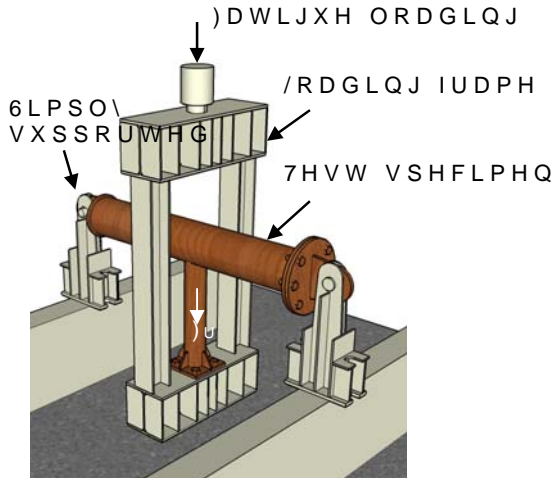
7DEOH ([SHULPHQWDO 6&) DQG )DWLJXH /LIH RI 7H

-RLQW	0D[LPXP 6&)	[LDO IRUFH) UDQJH]	DWLJXH OLIH 1
&)&+6			
&)&+6			
&)&+6			
&)&+6			
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&)&+6			
&)&+6			

&RQVWDQW DPSOLWXGH IDWLJXH ORKGV D WLHFV W DZ/HULF FR  
 ZHOGHG &)&+6 7 MRLQWV 7KH EWB FDQ B[L B D FFK F D LNF RIRQ V  
 IUHTXHQF\ RI +] 7KH VWUHV V UH P L Q L P G H I L Q H G D W R W R D I L P D V  
 ZDV IURP WR 6HHLQJ WK D W H W W K L Q D P V X B W Q R H F R R X V I G H R I C  
 WKH EUDFH ZDV UHYHUVHG DQG D D R Q G W K H I E W D P H H E H Q G Z H Z D Q V  
 WKDW D 3SXVK' IURP WKH IDWLJXH L Q W H R V L C S X O D F W L R H W K B V E W D  
 LQ )LJXUH 7KH DSSOLW H R S U D H I D F O I R R L F Q W U D Q J H Y H Q L Q 7 D E O H

\$ W \ S L F D O G D P D J H S D W W H U Q Z L W H O G D W R H X R I F F K R F U N G L Q L R V L O W L  
 &)&+6 7 MRLQWV LV VKRZQ LQ )LJXUH ZHG7 WKH D D W I D J X H J X H V F W D  
 LQLWLDWHG DW RU QH DU WKH FKR W K H S R Z Q V O L R F O R L P O [ Z R K P  
 VWUHV V L H W K H S R V L W L R Q R I B D [ L Q G I P F D W H I G V E \ F R Q D F M Q F W W I  
 SURSDJDWHG DORQJ WKH ZHOG WR HD Q Q W H Q D B D W L D Q R W J K H W K  
 FLUFXPIHUHQFH 7KH IDWLJXH OLIHG 1 I U R R F H D F K H & D W 1 6 J X H M R L Q W

7DEOH ZKLFK ZLOO EH XVHG WR XFLHQFN QW R H UIDFADLQ XHQ DQ M L  
 WKH WKHRU\ RI IUDFWXUH PHFKDQW K W Q & P V K IUV RS DSRIDG 1 FL 700  
 IDWLJXH LQGXFHG FUDFN IXOO\ SHHQ Y W H I D W H W X E X O H D D D O K R W K  
 FRPPRQO\ DFFHSWHG IDWLJXH IDLW KH I D W L L W H H L W E V W @ U H T X H  
 UHSRUWHG LQ WKH UHIHUHQFH > @



) L J X U H ) D W L J X H 7 H V W 6 H W W L X S H & U D F N

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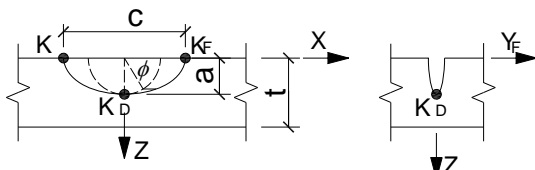
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 LQFOXGHV DVVXPSWLRQV IRU IDWLJXH DQ DQ FUDFN IUDPHWLRQ IDQ  
 PRGHO 7KLV QXPHULFDLQ DQ DQ V L V X E D W L Q J Z W K O H E W W X M M G L O O  
 RI & ) + 6 7 M R L Q W V

\$ V V X P S W L R Q V I R U ) D W L J X H & U D F N / R F D W L R Q D Q G 6 K D S I

, Q R U G H U W R I D F L O L W D W H Q X P H U D E R D W D W X O \ V Q V W D D O X P L S D F L  
 D Q G F U D F N V K D S H R I W K H Z H O G H G & ) & Q W D M R G L W D E D R M H G R R A Q  
 IDWLJXH VWUHQQJWK DQ G R E V H U Y H G I D W L J X H E H K D Y L R U

3.1.1 Initial crack location and size

7KH VXUIDFH FUDFN LV DVVXPHG W R L R Q L Z K I D F W K H K D W W K H F R L U  
 FRQFHQWUDWLRQ IDFWRU 7KLV ILQIDVH R I O H I P H L Q M G E D W K L G R S D K U  
 RI WKH & ) + 6 7 M R L Q W V F K R U G V X I E M R P F W L W D F O F F O R L B G O R G D S U  
 DVVXPHG W K D W W K H G H S H V W S R I D F W R V O R I F D W H P G E D W O W S W L  
 WKH F K R U G D V V K R Z Q L Q ) L J X U H



) L J X U H 6 F K H P D W D E S V R L F O P 6 X U I D F H & U D F N O R G H C

,Q WKH +6( GRFXPHQW > @ VODJHL QWRUPXVLRQV DUHPV SLOFGHLS  
W\SLFDO VLJHV DW WKH ZHOOPWRQ DWKHLWVW XGA WKH LQLWLD  
DVVXPALGLDV PP IRU WOKHLSWEEDO FUDFN VKDSH

3.1.2 Crack shape

%DVHG RQ WKH NQRZOHGJH RI OLQIRIUV HJODHGWVW SILUDDFOW &DIP DP  
IRU WKH ZHOHG &)+6 7 MRLQWV (FNJLV)LDVXUXPHG WRHHVVK  
VHPL HOOLSWLFDQ &KDOGH ZHOOPWRQ XGHS WKKRIZHOG WRH :KHQ X  
SURMHFWHG LQWR D SODQH WKH HOFOLSVELFDXO HGT XDRW GRQ FRU I  
VKDSH DV VKRZQ LQ )LJXUH 7KH GLUHFFHWLBRZWK DQVWPKHH  
SHUSHQGLFXODU WR WKH VXUIDFHROHWKXWFBRRBQEVH RD XHQH U  
SURSDJDWLRQ ,Q WKLW VWXG\ DWZGDWW B FLDVWLHQV ZIQW K WVKID FG  
DQG WKH FUDFN HQGV UHVSHFWLYHOQ WKH FDOFXODWRIZQ KEHF

)LQLWH (OHPHQW \$QDO\VLV

7KH PDMRU FKDOOHQJHV HQFRXQWHUM &HLQQWPKH BULFFDHOV PR  
ZHOHG &)+6 7 MRLQWV ZLWK VXUIDFH SDEWLQRVHFLQ FDXG  
PRGHOLQJ FUDFN PRGHOLQJ QRQOLQHO DWKQW HDQGFHR QOHFW  
DFRPSOLVK WKLW XVLQJ WKH JHQHUDO@FRVPSWSDWLRQID@6V  
3DUDPHWULF 'HVLJQ /DQJXDJH ZDVPFRSDR\HGRPHWKLW VWXG  
VFKPH DV ZHOODV FRQVWUXFWLQJ KDQJZK FJHDFPH WUSFVWQJX  
WKH FRPSOH[ SUREOHP RI PRGHOLQJDFHOGHDF &)+6HGF MRLVQW  
LQWR D UHODWLYHO\ VLPSON PRGHORUDRQGHG ZHOXWWRSIODW

3.2.1 Modeling procedure

,Q WKLW VWXG\ WKH PHVK RI WKH FZHOFGHGW ZHXOGV VSRHDV B VZ  
XVLQJ WKH \$3' / FRGH IRU D JRRG EDU DWKPH FEHDF ZHW QSU H RQGH  
DQG FRPSDWLELOLW\ RI WKH PHVK DQGHFRIRPSXW HGH OWHLPHQW S  
SURFHGXUH LV JLYHQ LQ )LJXUHV SDRIGHGK WIDLOUH RG HVDFFUL B/HG



D 7 EXWW ZLWK VXUIDFH FUDFN &+6 BQDEFWZUWRK WKH EXQD  
VXUIDFH RI D TXDUWHU RI



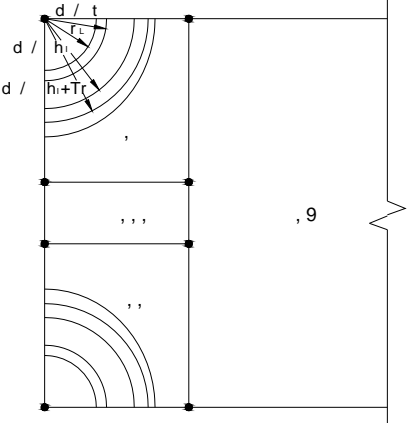
G 4 XDUWHU &+6 MRLQW H LQILOOHLQ 4XBU  
)LJXUH ,OOXVWUDWLRQ RI )LQLWH (OHPHQW ORGH

67(3 7KH ILUVW VWHS LQ WKH PRGHHGQJ ESXURWH\$0 DLVH V  
 VHPL HOOLSWLFD0 FUDFN GHILQH G DQZK HGLH/HQ K HRFKRUG/  
 ZDOO WKLNFQHVV &UDFN WLS VLQ0D0D0HULVORHQ00Q WRVU DDU FXID  
 EUDFH PHPEHU LQ WKH 7 MRLQW  
 67(3 7KH 7 EXWW SODWH LV WKH Q GE XELDWF HL QMRE HD ETHFDUKW  
 V\PPHWU\ RI WKH 7 MRLQW DQG DSSOLHG ORDG  
 67(3 \$ IODW SODWH PRGHO IRU WRQ0QHFRAUHG LW RJ HVKXHU DXD  
 PRGHO IURP 67(3 E\ PHUJLQJ WKH F S0PDRQH QDRGGH WRH H ERV K  
 67(3 7KH FKRUG SODWH PRGHO FLX00DXL 0\W ELQ ZLRW W KVK KIDS0  
 $\beta \gamma \tau t$  RI WKH MRLQW GHILQH G  
 67(3 &RQFUHWH HOHPHQWV DUH D0 G M GH [HRJ FV K/HH G KKRUGH PV  
 WKH LQWHUIDFH PHVK RI WKH FRQ0W0DWW B Q 0 PVH0MHVODUKR 0  
 LQWHUIDFH RI WKHVH WZR PDWHULDOV

3.2.2 Geometric transformation

\*HRPHWULF WUDQVIRUPDWLRQ LV H W S 0 G DQG KHRJ HF LW R XPDSU QR  
 PDLQWDLQLQJ WKHLU WRSRORJLFDOW V H 0 D R V L R Q V K E SW UIDRQV IV  
 IRUPXODV KDYH EHHQ GHULYHG DQG LP SIOQUP WQW H S ' L Q W R P W Q  
 LQ WKLV VWXG\ &UDFN WLS VLQJXLOFKULV OSHPHHQLVR XVD0A EHRHQ D  
 WKH \$16<6 HOHPHQW OLEUDU\ , Q RJD0HLR QV B DWLKP X 0 D W K I W W K H H  
 MRLQW VLJHV KDYH EHHQ FKR VH Q 0 D 0 W K H Q Y R D Q 0 L Q J H D S U F R R H W  
 EHWZHHQ WKH VWHHO FKRUG PHPHPS 0 R Q G G V K H F R Q F U H W H L Q

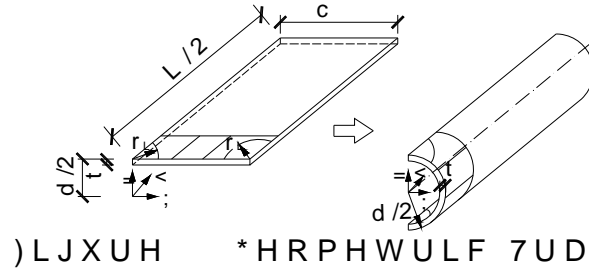
)RU KLJK TXDOLW\ PHVK JHQHUDW URVH UWRKIHF X Q R G 0 B 0 G 0 X Y L 0 G F  
 UHJLRQV DV VKRZQ LQ )LJXUH R Q F H H Q W R Q F , 0 I Q G F 0 , H V D Z L W U K L 0  
 r L KDYH EHHQ 0 U D Z I Q U Z K S U H V H Q W V W K H E U D F E H 0 Q 0 H U Z D O O  
 UHSUHVHQWV WKH FKRUG L Z H 0 G 0 F W R H H S W R I M H Q W W L R 0 K 0 C E R U G H U  
 GLIIHUHQW WUDQW ZLFDOW ERQXMRH0H0D QW K R I X W Q H 0 H 0 R 0 R I V R Q L  
 hr LV WKH ZHOG WRH K H I L J K W K R Q Z W O G H W F K R 0 0 L V K 0 V H R Q W K H  
 IRU ERWK 5HJLRQ , DQG , , QHHGV WR EH V\PPHWULFD0



)LJXUH )RXU 5HJLRQV IRU )LQLW 0 X (0 I H D F H H Q R M 0 H 4 / K D R 0 W H V K I R 18  
 , W LV QRWHZRUK\ WKDW WKH Q R V G H H O R Q X V I K H D Q G W H I C H L R U R 0 H V  
 VXUIDFH RI WKH FRQFUHWH FRLQF0L0R0M E H W P H H D H F 0 W W K H W K Z K L  
 LW PD\ OHDG WR Q R I 0 U F R Q 0 Y R I X V H 0 W W H K 0 W 0 V F L Q Q W K V L P L X 0 L D W L R Q  
 \$16<6 PRGHO GXH WR WKH LQLWLD0 Z J D S F R Q W 0 F 0 W H 0 W 0 D 0 M U L I R 0  
 I R O O R Z L Q J I R U P X 0 P D W 0 D W K 0 H U H R Y H 0 W L R I F H 0 D 0 L 0 V I R U P D

3.2.2.1 Geometric transformation for chord

\$Q XQIROGHG SODQH VXUIDFH PHVK (R) LDJ TXH U WLN UWRUIDQK/R B G P  
FXUYHG VXUIDFH PHVK XVLQJ (TV DQG



)LJXUH \*HRPHWULF 7UDQVIRUPDWLRQ IRU &KRU

)R  $X \leq d$  hf Tr DQG  $d$  hf Tr VHH )LJXUH

$$\left\{ \begin{array}{l} X = ZVLQ \rightarrow \frac{d \text{ DUFVLQ} - \frac{X}{d} \frac{d}{t} \frac{-Z c - c d}{c}}{Z} \quad @ \\ Y = Y \\ Z = ZFRV \rightarrow \frac{d \text{ DUFVLQ} - \frac{X}{d} \frac{d}{t} \frac{-Z c - c d}{c}}{Z} \quad @ \end{array} \right.$$

:KH@HLV WKH RXWHU FLUFX@IHUVH@FH LRQ QH @KRLWG X@QJHQFH R

)R  $X \geq c - d$  hf Tr) DQG  $d$  hf Tr,

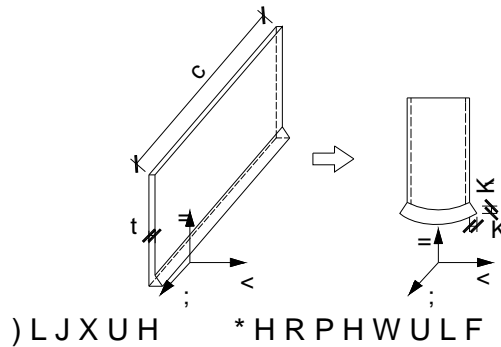
$$\left\{ \begin{array}{l} X = ZVLQ \rightarrow \frac{c - d \frac{c - X}{d} \frac{d}{t} \frac{-Z c - c d}{c} - \frac{c - X}{d} \frac{d}{t} \frac{-Z c - c d}{c}}{Z} \quad @ \\ Y = Y \\ Z = ZFRV \rightarrow \frac{c - d \frac{c - X}{d} \frac{d}{t} \frac{-Z c - c d}{c} - \frac{c - X}{d} \frac{d}{t} \frac{-Z c - c d}{c}}{Z} \quad @ \end{array} \right.$$

)RU RWKHUV

$$\left\{ \begin{array}{l} X = ZVLQ \rightarrow \frac{X - \frac{d}{t} \frac{-Z c - c d}{c}}{Z} \quad @ \\ Y = Y \\ Z = ZFRV \rightarrow \frac{X - \frac{d}{t} \frac{-Z c - c d}{c}}{Z} \quad @ \end{array} \right.$$

3.2.2.2 Geometric transformation for brace

\$ SODWH PHVK ZLWK ZHOG LQ )LJXUPHV K VXWUDQJ (TRUP HQG G QW



) R ∈ d > d + h\_f @

$$\begin{cases} X = YVLQ \rightarrow \frac{X - \frac{d + h_f - Y c - c}{t + h_f c} X}{Y} @ \\ Y = YFRV \rightarrow \frac{X - \frac{d + h_f - Y c - c}{t + h_f c} X}{Y} @ \\ Z = Z - d > -\sqrt{d - X} @ \end{cases}$$

: KHHLV WKH LQQHU FLUFXPHVHQH SURMEHFDLHR QDQG QJWK LG RQ WKH FKRUG VLGH

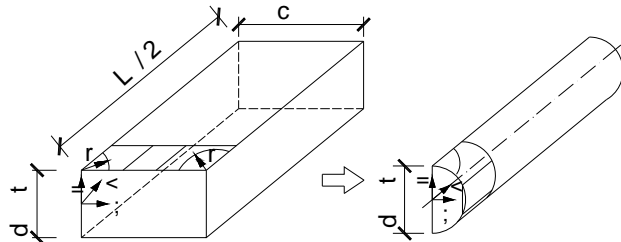
) R ∈ d > +h\_f d + L @

$$\begin{cases} X = YVLQ \rightarrow \frac{X - \frac{d + h_f - Y c - c}{t + h_f c} X}{Y} @ \\ Y = YFRV \rightarrow \frac{X - \frac{d + h_f - Y c - c}{t + h_f c} X}{Y} @ \\ Z = Z - \frac{d + L - Z}{L - h_f} d > -\sqrt{d - X} @ \end{cases}$$

3.2.2.3 Geometric transformation for concrete

, Q RUGHU WR PDNH WKH QRGHV RQ XVEKHEHQ WKH LVRUP MXZUWKHW WKH H[WHUQDO VXUIDFH FRQFUHWH QJH (TK/LQ) LDXGH LV WUD





) L J X U H \* H R P H W U L F 7 U D Q V I R U P D W L R Q I R U & R Q F U

) R  $\alpha \leq d$   $h_f$  Tr D Q G  $h_f$  Tr

$$\begin{cases} X = d - t - \frac{c d \sqrt{L Q}}{c d - t} \\ Y = Y \\ Z = \frac{Z \sqrt{d - t - X}}{d - t} \end{cases} @$$

) R  $\alpha \geq c - d$   $h_f$  Tr D Q G  $h_f$  Tr,

$$\begin{cases} X = d - t - \frac{c - d \arccos \frac{c - X}{d}}{c d - t} \\ Y = Y \\ Z = \frac{Z \sqrt{d - t - X}}{d - t} \end{cases} @$$

) R U R W K H U V

$$\begin{cases} X = d - t - \frac{c X}{c d - t} \\ Y = Y \\ Z = \frac{Z \sqrt{d - t - X}}{d - t} \end{cases} @$$

### 3.2.3 Material property and element type

, Q W K H I L Q L W H H O H P H Q W D Q D O V L P X O B R / H G S D U @ J R W K H K 6 2 V, W H I  
W K H \$ 16 < 6 V R I W Z D U H F H I F H S W U S U V W K H O P R G X O X H P R Q M O D V W L I  
3 R L V V R Q 1 V U D W L R D U H V H W \* 3 D D Q G > @ U H V S H F W L Y H

3 D U W L F X O D U D W W H Q W L R Q Z D V J L Y H R Q W R U N O G H L Q W L D Q H G W K K H  
I D W L J X H O R D G D V W K L V L V L P S R Q V D Q K X W R D W K J H X H D W W J X H Q J D  
I D W L J X H W H V W Z D W X R P S O H D W H G K N W K I R H O U D V F H F D Q G Q W R K H F K R U  
H [ S R V H W K H F R Q F U H W H I R U D F O R V H H P U P I D Q M W S I R F O O \$ V S H F R Z

YLVXDO VLJQ RI FRQFUHWH FUXVKLQJRZKDG V H R E V @ E D T X H S Q  
HOHPHQW DQDO\VLV WKDW WKH F R Q F U H W H G L V W I L L O E X W K B Q J D W V  
D & +6 EUDFH DQG D & +6 FKRUG DQGWV R X V D D Z H F G V H G W K R L Q D W I  
ZKHQ FRQFUHWH VWUHQJWK JUDGH D L F K D O J R W W W W I K H R V P V S R S O  
HIIHFW RI FRQFUHWH VWUHQJWK S U R D S H U W M B Q W L K B L F R Q F O W W E  
EH QHJOHFHGH 7KH IDWLJXH GHIREU P D R X L R Q I P R O M O K & K R O H F R  
the Design of Concrete Structures \* % 7KH\ DUH VXPPDULJHG LQ  
3RLVVRQ TV UDWLR RI FRQFUHWH L H W H O H P H Q W Q D W P R G H Q W F R R Q  
VLPXODWHG XVLQJ WKH 62/, ' HOHPHQW



D & RQFUHWH H I Q W L L R P H I L E H U & R Q E V H U W L Q H F R P S H U H V I L R Q  
)LJXUH 9LHZ RI & RQFUHWH , Q I L O O ' D P D J H V

7DEOH & RQFUHWH )DWLJXH 'HIRUPDWLRQ ORGX  
& RQFUHWH JUDGH & & & & & &  
03D

7KH ZHOG LV PRGHOHG XVLQJ 62/, ' H O H P H Q W U L Q F W Q H L J S U B  
GHILQH DFFRUGLQJ WR WKH \$:6 H U D I O R S P R S Q G D W L L R Q Z H D Q  
WR EH WKH VDPH DV WKH SDUHGW VWHHO PHWDO

7KH FRQWDFW HOHPHQWV 7\$5\*( D Q G Z D H 7 \$ D W V L Q H W L K Q W \$ H U  
FRQFUHWH LQILOO DQG VWHHO F K R K I G F R X E M D Z M U F H F K S O R W F G E  
VWHHO DQG FRQFUHWH F Z K Q W U I Z D W W K H W L Q W H U I D F H

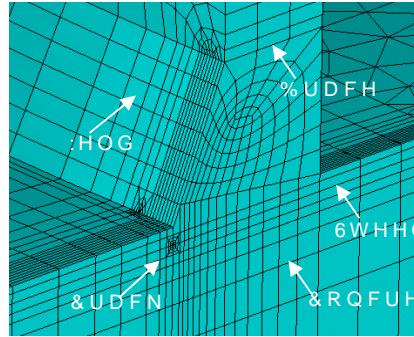
7KH FUDFN WLS VLQJXODU HOHPHQW X P M H U H L V M D B F K > L V @ E D K  
PLGGOH QRGHV LQ WKH ZHGJH VKDSH I G U H S R L F H Q W R I D W I K H P R O H I K  
RUGHU WR R R G H L Q J W O B U L W \ 6 L Q F H W K L V W I S H B I M O K H P H Q W  
ZHGJH VKDSHG VLQJXODU HOHPHQW Z B O H P R O W W U X X E V H G W K U R I  
SURJUDPPLQJ ODQJXDJH



)LJXUH &UDFN 7LS 6LQJXODU (OHPHQW

,Q RUGHU WR DFFXUDWHO\ VLPXODWH R Q F U H W H B B Q G R Q J V L Q U M K M  
UHJLRQ PRUH WKDQ IRXU OD\HUVJ RW K H Q L K R H U G I Z H P O Q W K L Z H N Q H  
DQG W K U H H O D \ H U Z D O O R Q J L F N K H E K U S E M U H F D W I F R Q O Z K I L F J H I R U W  
EHWZHHQ WKH ILQH PHVK QHDU W K H P H L Q W H U K H F W D R Q W Q S W K

E\ IRXU ULQJV RI HOHPHQWV DQG H)LFK UHLQJ VFRZWDQGV RI VHK  
 ILQLWH HOHPHQW PHVK DW WKH FKRUG FURZQ SRVLWLRQ



)LJXUH &ORVH XS 9LHZ RI )LQLWM &ORZDQSRVH WKROHDU

675(66 ,17(16,7< )\$&725 &\$/&8/\$7,21

\*HQHUDO 0HWKRGRORJ\

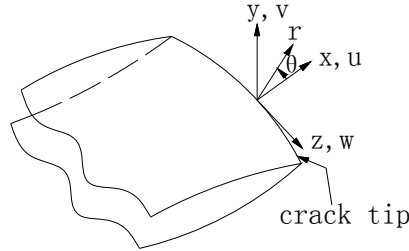
)URP WKH NQRZOHGJH RI OLQHDU HEDMWWF IDFW VWKH PHUFXR  
 VWUHV V LQWHQVLW\ IDFWRUV FDQ EH H[SUHVVHG DV IROORZV

$$\begin{cases} u = \frac{K}{*} \sqrt{\frac{r}{OE}} N \frac{\theta}{FRV} FRV @ \\ \frac{K}{*} \sqrt{\frac{r}{OE}} > N^{\theta} VLQ r VLQ @ R \\ v = \frac{K}{*} \sqrt{\frac{r}{OE}} N \frac{\theta}{VLQ} VLQ @ \\ \frac{K}{*} \sqrt{\frac{r}{OE}} > N^{\theta} FRV r FRV @ R \\ w = -\frac{K}{*} \sqrt{\frac{r}{OE}} VLQ^{\theta} r R \end{cases}$$

ZKHUH

$$N = \begin{cases} -\nu & \text{plane strain} \\ \frac{3\nu}{1+\nu} & \text{plane stress} \end{cases}$$

,Q (Tu v w GHQRWHV WKH ORFDO UDGLDO GRUPLDQJ WDJH  
 K,,, DUH WKH PRGH , , , , , VWUHV\* LQWHQVLW\ WKH IDFW  
 GHQRWHV WKH 3RLVVRQ V UDWLR

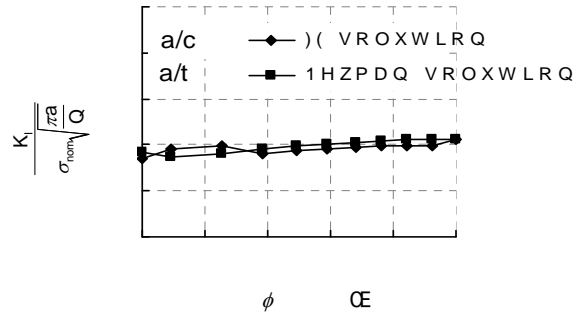
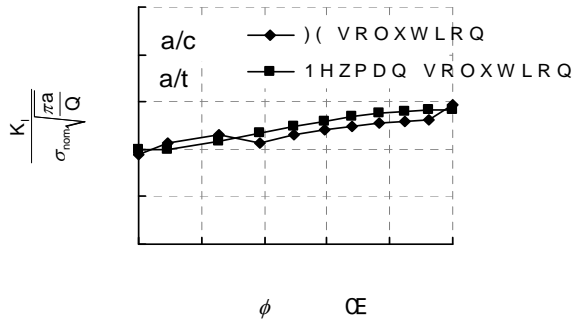


)LJXUH &UDFN 7LS &RRUGLQDWH 6\ VWHP

%DVHG RQ WKHVH HTXDWLRQV GLVSO@F Z B M QHP S Q W UHDS RWRD FV  
 WKH VWUHV LQWHQVLW\ IDFWRU S E W K D L Q G H V K E G H L Q W Q J H D D O  
 WKH FUDFN PHHWV WKH ZHOG \$ KR B B Q Y D V X H Y V R U X Q A K H S O H D C  
 VWDWH DUH JUHDWHU WKDQ WKRE\H YDSXUHWK\ QURPZ\ W V K H S O D R H H  
 YDOXH 3ODQH VWUDLQ VWDWH ZDV) R U V W X P H G D E N H % R I Z Q R Q V H U Q  
 DQ G V L P S O L F L W \ S O D Q H V W U D L Q D V V X P S W L R Q L V D G R S W H G I

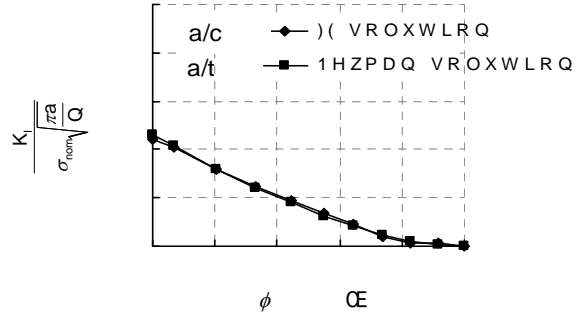
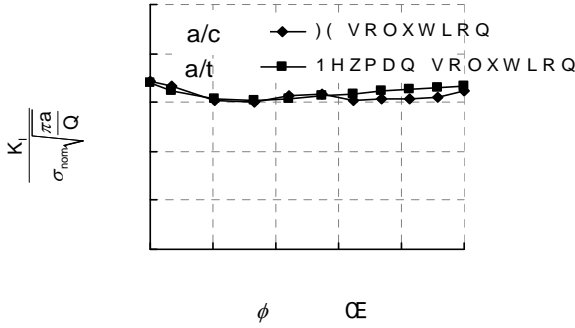
9HULILFDWLRQ ZLWK )ODW 3ODWH

7R YDOLGDWH WKH \$3' SURJUDPV DQ G W R K O X F U L L R F W V R I L S W K I H Q V X  
 IDFWRUV IRU D V S P S O W H F Z L W K G H O D W H Q W R P S X F W H G L Q Q G F R P S D  
 ZLGHQ\ DFFHSWHG UHVXOWV JLYHQ E H 1H Z P D Q S O D Q G I R D M W K W Z  
 PRGHO LV FRQVLVWHQW ZLWK WKH ZIURQMG & K & + 6 O D M R I L Q D W  
 ZLGWK DQ G W K L F N Q H V V R I P P I P I R O O Q R Z L R P J W H Z V S I R W D G  
 ZHUH FRQVLGHUH G W K H I L U V W R Q H Q L D V O V P H H Q E L U D I Q H O R D V G J W V H W X  
 WKH RWKHU RQH LV EHQGLQJ PRPHQW O U B V L X O W W K O J H L Q W H O R P I L L  
 I R O O R Z L Q J F U D F N V L J H V D U H F R a Q V L G H U R H G L Q W K H Y D O L G D W L  
 7K H F R P S X W D W L R Q K U B Q S O W H V K R Z W W K D W K R Q H K S V D U F H Q W D J H F  
 F D Q E H Q H J O H F W H G Q R V 7 K H Y D O W K W H F U D F N W L S I U R P W K H I L Q L V  
 F R P S D U H G Z L W K W K H 1 H Z P D Q 5 D M X 1 W V R O X W L R Q U H R U E B Q C  
 F U D F N V V H H ) L J X U H , W F D Q E K I D W H W Q H U R P H Q G X U D J U H H D Z  
 R W K H U σ<sub>ZR</sub> H U V W W K H Q R P L Q D O Q V W U D H V X Q F O V L R Q W K H H O  
 1 H Z P D Q 5 D M X 1 V V R O X W L R Q 7 K H P D [ L P X I P V G L O W U H L S D Q F R F E F W U Z  
 W K H G H H S H V W S R L Q W D Q G D W H W H O H F H Q W N P R O G G O R D Q S W O R C  
 6,) U H V X O W V L Q W K H F D V H R I D I O D W S O D W H Z L W K V X U I D F H F



D ) O D W S O D W H L Q W H Q W R S Q U H E H Q G L Q J P R F

)LJXUH, 'LVWULEXWLRQ RI D )ODW 3ODWH ZLWK 6KDC

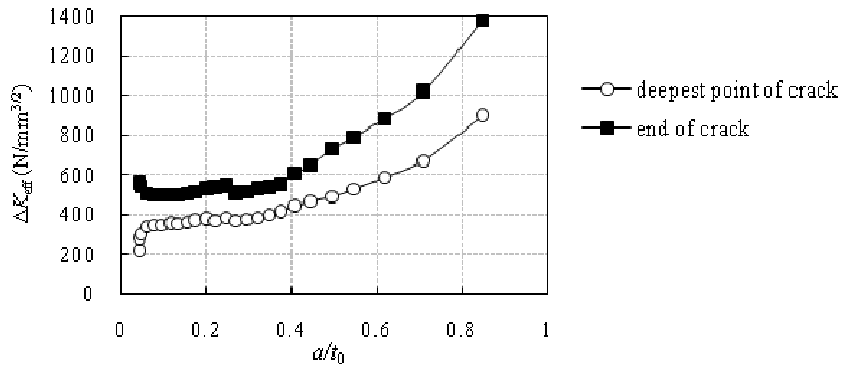


D ) O D W S O D W H L Q W H Q V I S K Q H E H Q G L Q J P R P H  
 ) L K J X L W M U L E X W L R Q R I D ) O D W 3 O D W H Z L W K ' H

& D O F X O D W L Q J 6 , ) V I R U : H O G H G & ) & + 6 7 M R L Q W

\$ I W H U W K H I L Q L W H H O H P H Q W P R O M I O E L V V Y D O L I G D W W G H L Q / D L Q W  
 W K H Z H O G H G & ) & + 6 7 M R L Q W V Z L W K V X U I D Q H L W U H D F N Q H Z P H J O  
 6 S H F L P H Q & ) & + 6 7 M R L Q W L Q W K H K R W W S R W \ V : D U C H V H M [ S D I  
 7 R Q J M L 8 Q L Y H U V L W \ L V V H O H F W H I G I R D Q W K I D E O M X G Y K B V G L R R Z  
 W K L V 7 M R L Q W V S H F L P H Q D U H P P  $\alpha$  F R Q F U H W H J U D G H L V  
 V W H H O P D W H U L D O L V 4 F U R S I Q D N L I D O D M R D S I S O L D H G H W R W K H E U I  
 7 M R L Q W 7 K H F D O F X O D W L Q J 6 , ) V I R U : H O G H G & ) & + 6 7 M R L Q W  
 W K H P L V W K X V Q H J O L J L E O H Z K L F K R Q W H G L M W X U G H Q R Q I S D P W K H  
 H I I H F W L Y H V W U H K A V K I D Q W E I H O H Q W X V H G F W R U H Z K L F K F D Q E H F R P S  
 D V J L Y H Q L Q % 6 > K @ H I D , Q G W M K U W V W X G M Q W R G H Q R W H W K H  
 L Q W H Q V L W \ I D F W S B L I Q W D Q G G K H H S H V W F I N O H Q Z K L F K H K S Y H E H H G  
 I R U G L I I H U H Q W F U D F N V L J H V D V S U H V H Q W H G L Q ) L J X U H

$$K_{HI} = \sqrt{K_1 + K_2 + K_3 \dots - v^2}$$



) L J X U H 9 D U L D W L R Q R I ( I I H F W L Y H 6 W U H V V , Q W H  
 D W W K H ' H H S H V W 3 R L Q W D Q G & U D F N ( Q G V G X U L Q J

) \$ 7 , \* 8 ( / , ) ( 0 2 ' ( / , 1 \* \$ 1 ' 3 5 ( ' , & 7 , 2 1

\$ F F X U D W H I D W L J X H O L I H S U H G L F W L R Q G L D S I O G V W Q R W V R Q O W H  
 V R O X W L R Q E X W D O V R R Q W K H D S S U R S U L D W H F U D F N J U R Z W K

)DWLJXH \*URZWK ORGHO

3DULV /DZ ZKLFK LV WKH PRVW SROS XIODUWID Q WJXHG K E B G NEJ  
(UGRJDQ > @ UHODWHV WKH IDWVJ K Q WFHUVFVWJ UI B ZWR U DW B J  
(T E HORZ

$$\frac{a}{b} \& K \hat{u}^p$$

ZKHUHV WKH QXPEHU RI VORFD 6 KDSH \$VVXPSWLRQ WUHVV LQWHQVLWV  
DQG P DUH WKH FUDFN JURZWK PDWQLHUHQV WR R QVDFV Q V D W HWWLK  
E\ LQWHJUDWLQJ (T LI WKH K V W WKVHV LI Q W H Q V L F D I E N F W R H S V  
ILQDO FUDFN DGH S W R Z Q L H

$$\int^1 NG \int_a^{a_1} \frac{a}{\& K^p}$$

+RZHYHU GLILFXOW\ H[LVWV LQ R E W E D L F O L X Q J H W K H U H Y S O U F I G W  
RFFXUV ZLWK JURZLQJ FUDFN VL]HO FR B S E H D Q O G H G V & K B + 6 G  
3DULV /DZ LV H[SUHVVHG LQ DQ R K G L Q B Q \ L Q L I M H L D Q Q W L O X H H T  
(XOHU PHWKRQ FDQ EH XVHG WR FRQYHUW (T LQWR

$$N_q \quad N_q \frac{a}{\& K_q^p} \quad Q \quad W$$

ZKHUHV WKH QXPEHU RI LQWHJUDWLQJ W K E L Q M D H U Y D O X H & V S E  
3' > @ & L H x - D Q G P

&UDFN 6KDSH \$VVXPSWLRQ

7KH 1HZPDQ DQG 5DMX V 37ZR SRLQW 6 E X L Q W V P W K H O O H S V L H H O P H S V  
UHODWLRQVKLS RI WKH FUDFN V D F H Q J W K S H Q G V G D H S W X P H T K W R H E R U B  
WKLV VWXG\ DQG WKH FUDFN JURZV E \ L Q K W K 3 D W L Z R / G Z U H F W H D Q Z L V

$$\left\{ \begin{array}{l} \frac{a}{b} \& K \hat{u}^p \quad D \\ \frac{a}{b} \& K \hat{u}^p \quad F \end{array} \right.$$

ZKHUHV D Q G F G H Q R W H W K H V W U H V V L Q W H Q V L O H Q D W W R G L U H D Q W E  
UHVSHFWLW\ H G W H H U P L Q H G I U R P H U S E D W H D Q G W L Q & W K H S D V W G I  
UHODWLRQV KDYH EHHQ S U B S R V H G & S I R U Q H W K R S O V H & X L G V &  
DGRSWHG VLQFH LW LV PRUH FR Q V L Q U Y D W D W L R Z K L F K O H D G V W

$$\left\{ \begin{array}{l} a_q \quad a_q \quad N \quad K \hat{u}_D^p \quad H I I > \quad \& \quad \hat{u} \quad @ \\ c_q \quad c_q \quad N \quad K \hat{u}_F^p \quad H I I > \quad \& \quad \hat{u} \quad @ \end{array} \right.$$

)DLOXUH &ULWHULRQ DQG )DWLJXH /LIH (VWLDPDWLRQ

7KH FULWLFDO FUDFN VL]H LV GHILQHG IDW WKHH FOLDFEN VLWHD W  
DVVXPHG WR RFFXU )RU WKH IDWRJXR ZLVHFMLRQP DWLROQV  
GHWHUPLQHG HLWKHU DV D FULWLFDO OHLQJWK YRU\DMH Z UH  
DYDLODEOH IRU ZHOGHG &)+6 MRLQWV FWKWHIRIODO RZHSRJJ WUHL  
KROORZ VHFWRQ MRLQWV PLJKW SHRIVLGHJXLQV DJLQWURIQF KRV  
ZHOGHG &)+6 7 MRLQWV LQ WHZBOORWKUENQLFDO DVU DUFNWV  
XVHG E\ 6FKXPDKHU HW DO > @WLADLO FUDZDNO VYJKLFWHQH V  
1XVVEDXPHU > @ DQG 1XVVEDXPHU DQGO+ZDGO PDKQF NQMV  
FRPPRQ\ DFFHSWHG DV WKH FULWLFDO [VHUQVINYHQ]HLGHWFK  
GLUHFWRQ ZKHQ WKH FUDFN SHOHUWUDULHD WKRQIKO OZDVOVW  
IDWLJXH OLIH DV WKH PDMRULW\GRHSWKH OLIH LV VSHQW DWV

7R DYRLG XQH[SHFWHG IDLOXUH SHRSOHU XDWDOE\YDQXHI WKI  
VHUFLFHDELQW\ FULWHULRQ DQ\WKKHFWXWQ FDKLFNQFN WRJHW  
WDNHQ DV WKH FULWLFDO FUDFN VL]H

7KH IDWLJXH WHVW UHVXOWV RI WKKHRZDQDGGW&)+6 BRQ MORLQW  
VKRZ WKDW WKH ORDG F\FOHV FRGHQWSRLQGLQJ DFN WKLDFNU  
WKURXJK WKH ZDOO WKLFNQHVV RSHFKLPHQV MHORW XERIOD BDF  
FSDFLW\ GLIIHU IURP HDFK RWKHU VFKLQHVW XGKDOVKH VLKX  
WHUPLQDWHG ZKHQ WKH FUDFN GHVSWK\ ULQDFKFDW MCKHLQKBJXG

5HVXOWV DQG 'LVFXVVLRQV

)ROORZLQJ WKH DIRH BKHUHLRQW GSKUR &)+6 RMRQW ZM DMH  
F\FOLF WHQVLRQ ORDG LQ LWV EUJDFH FZDQ\EGHYBORSGVHGW  
WR LPSOHPHQW WKLW SURFHGXUH LQ V&K+\$1676MRLQWZ DLSHF\$  
7DEOHL ZLWKPP DQG PP ZDV OLVWHG WR LOOXVWUDWH WKH  
GLYLGHG LQWR D WRWDO RI VDVH SVK RZRQ GLQWHDUEFLQH WKH ID

7R HYDOXDWH WKH YDOLGLW\ RI MKKHDMDWIXJK OLOHIRIPRQOZIHQ  
7 MRLQWV ZHUH SUHGLFWHG DQG FOPSDWHDG ZLVWKYDQDLQDZDQ  
)LJXUH ,W FDQ EH VHHQ IURP )LDWUJXH WIKYHW WKHVRSDJHQ  
WKH H[SHULPHQWDO GDWD 7KH IURLOEXRZHLQVRI DFKWRGLVDFWHI SCLQ  
) ( UHVXOWV DQG H[SHULPHQWDO QULWLVDO FVWDFN WKLHVPJLYG  
ZDV XVHG KHUH ZKLFK PLJKW EH GJLHWHQW KIH RPH DQKHY DFXK  
DQG P SDUDPHWHUV VHW E\ WKH 3'HRQO@ HPSLZUHLFOO DVPBGH  
XVHG WR DSSUR[LPDWH WKH IDWLJXH JLRZWKHRLQKDGHW&)+6  
ZHGLQJ SURFHVV ZDV QRW WDNHQWLQV RGHDFFRXQW[SLQWPK  
GHWHUPLQHG IDWLJXH OLIH PLJKW LQWRWDFWUZKHVS RQV LWXHWK  
WKH IXOO FKRUG WXEHWKLFNQHDWIGJXHWRU GEINLBNQWVWKLQ  
GLUHFWRQ

, Q S X W ' D W D  
 $a = L Q L W L D O F L E F I C L W H S V K F U D F N K D O I Z L G W K$   
 $a_1 = I L Q D O F U D I F N L G X L S W K U L W H U L R Q$   
 $\& = F U D F N J U R Z W K P D W H F U L D I F N F L Q R Z W D Q M S R Q F H Q W$   
 $\Delta N = L Q F U H P H Q W L Q W K H = Q X I F Q M U R R I U F D Q J H V$   
 - R L Q W J H R P H W U \beta S D U D P H W H U V  
 6 W H H O D Q G F R Q F U H W H P D W H U L D O S U R S H U W \ S D U D P H W H U

3 D U D P H W H U , Q L W L D O L J D W L R Q  
 $a \ a_L c \ c_L N$

& D O F X O D W H ' L P H Q V L R Q & D O F X O D W H 6 W U H V V , Q W H Q V L W \ ) D F W R U S I D Q K H  
 $) D F W R U \frac{a}{a_1} \frac{c}{c_1} N$

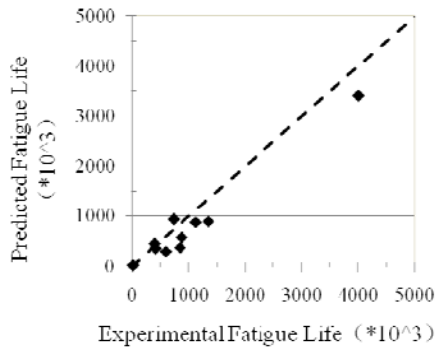
3 D U D P H W H U 8 S G D W L Q J  
 , )  $\Delta K_D$   
 $a = a + \Delta N > \& K_D^P$   
 , )  $\Delta K_F$   
 $c = c + \Delta N > \& K_F^P$   
 $N = N + \Delta N$

$\Delta K_D \leq$   
 $\Delta K_F \leq$   
 < H V

& U D F N L V 1 2 7 J U R Z L Q J

1 R  
 $a \geq a_1$   
 < H V  
 ) D W L J X N H / L I H

) L J X U H ) O R Z & K / L U H V ( R W ) L P W D W L X R H Q 6 R 7 M H P L Q H G V & ) & +



) L J X U H & R P S D U L V R Q E H W Z H H Q 3 U H Q L Y F W H G D Q G ([ S H



7DEOH		,OOXVWUDWLYH ([DP SOH&IRU )DWR LQW/LIH (VW						
VWHS	a	c	$\Delta K_D$	$\Delta K_F$	$\Delta N$	N	@	@
PP	PP	PP	1 PP	(1 PP)	î	î	PP	PP

7DEOH	&RQSDULVRQ LQ )DWLJXH /LIH EHWZHHQ )
-RLQW	)( SUHGLFWHG IDWLJXH OLIH ULPHQWDO IDWLJXH OLIH
&)&+6	
&)&+6	
&)&+6	
&)&+6	
&)&+6	
&)&+6	
&)&+6	
&)&+6	
&)&+6	
&)&+6	
&)&+6	

5.4.1 Effect of initial crack size

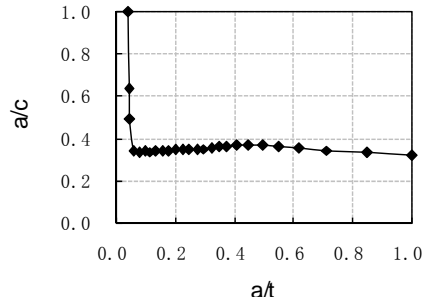
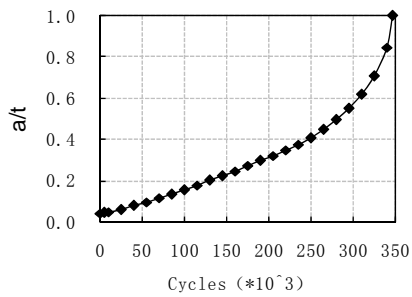
7KH GDWD SUHVHQWHG LQ 7DEOH VLVWHPDQGHG RQDPVV,XP HLG IRQ  
 LQWHUHVW WR VWXG\ WKH HIIHFWRU RI IDWLJXH OLIH H QWWEDWLRQDF  
 FDVHV ZLWK LQFUHDVHG LQLWLDO FUDPH VLLJH PDL  
 CL PP ZKLFK IDOO ZLWKLQ WKH VQ DJ @Q VHXWLFKRV VHQ JKH U  
 SDUDPHWULF VWXG\ RI WKH &)&+6 H WFDLQWHV SMKHPHQWQWQD  
 WKH ORQJHU WKH IDWLJXH OLIH L\$V VHXZQLQ DDIHDERXWKH  
 WKH EHQFKPDLNLF DVH PZLW, W LV IRXQG WKDW WKH IDWLJXH  
 7 MRLQWV ZLWK QRUPDO TXDOLW\ ZHQ@WKZQWKKHQVQDQD VSHDF  
 GR QRW FKDQJH PRUH WKDQ

7DEOH (IIHFW RI ,QLWLDO &UDQN 6L]H RQ )DWLJXH

1R	,QLWLDO FUDFN VL]H 1
a <sub>L</sub> c <sub>L</sub>	PP
a <sub>L</sub> c <sub>L</sub>	PP
a <sub>L</sub> c <sub>L</sub>	PP
a <sub>L</sub> c <sub>L</sub>	PP
a <sub>L</sub> c <sub>L</sub>	PP

5.4.2 Crack growth behavior

)LJXUH VKRZV WKEH WZHODV IFRDVKL SV FHOV D ORG WWHU H&) & +6 7 VSHFLPHQ ZKLOH RWKHU VSHFLPHQV HVIKIDV W KH VLPDQ D UWW UR IDWLJXH OLIH LV LQ WKH VKDOOR ZHF UIDFNLV W H JFU D QNV KD Q R GHSWK GLUHFWRQ ZKLOH SURSDJD W LHW LVR HMLGDHSLFHG DOWR ZKHQ WKH UDWLORIGURSSHG IURP WR NDV FUDFN GHSWK UHDFKHG KDOI ZDOO WKLFGHVQ ERUDK NG UHREZWKR Q REVHUVDWLRQ DJUHHV ZHOO ZLWK W KUDMISRO UPRQ WWH VIXOW WR ODUJH FUDFN VSUHDG RYHU WKRUFK RQGF ZIDOO LJX HDEUDF YLVLEOH UHWURILW DFWLRQV VXFKE HDV RQW HQDQD W KH QKRWF ORFDWLRQ RQ WKH &) & +6 VWUXFWXUH XQGHU IDWLJXH ORDGL



D & \a) & +6 HODWLRQ FXUYH E )LJXUH 5HODWLRQV KLS EHWZHHQ &UDFN 6L]H DQ

5.4.3 Effect of concrete grade

&) & +6 &) & +6 DQG &) & +6 7 MRLQWV KQRE FIRPHOHLRQOV JHRPHWULFDQ SDUJFKHWGLUWHUHQW FRQFUHWH JUDGHV H H G 7DEOH ,W FDQ EH VHHQ IURP 7DEQ SHUWFKDQV DQGH &) & +6 WKUHH &) & +6 7 MRLQWV UZLGHV FRQF R W DQ G Q L I L E D Q W DOWKR) GHFUHDVHV DV FRQFUHWH JUDGH DQFRUHDQV W) L D W W G W Q H P H H [ D P S O H I R U D 7 M R L Q W K D Y L Q J W I Z H W F D H F X Q H D M G L Q + W K H 6 F R Q F U H W H J U D G H V I U R P & W R & D D W K B Z G H E Q H D D E H O V H D V F R Q F U H W H J U D G H L Q F U H D V H V E X W W K X V X D G L O M H X I H Q G H F R R F S J F R Q F U H W H I L O O H G W X E X O D U V W U X F W X V H I H D W X U Z L V P K I L D Q W W K L L P S U R Y H P H Q W L Q I D W L J X H E H K D Y I R Q F U R H W X E X O V I L W Q M R L L E D Q W E O W K H U L J L G L W \ Q R W V W U H Q J W K R G L F R Q R U H M R L Q Z K L B Q G L F S R O F G H F U H D V H V K R U W V S R H W V F R Q F W H Q W U D W L R Q

7DEOH (IIHFW RI &RQFUHWH 7HUVHG RQ 6&)  
 -RLQW &)&+6 &)&+6 &)&+6  
 &RQFUHWH JUDGH & & &  
 6&DUDWLR WR &

7DEOH (IIHFW RI &RQFUHWH) (\$J DGH RQ 6&)  
 &RQFUHWH JUDGH & & & & & &  
 6&DUDWLR WR &

&21&/86,216

7KLV SDSHU RXWOLQHV D IDWLJXH GIGIRDFUHHWWPH HQVO BGRFHLU  
 VHFWRQRQ &)&+6 7 MRLQW VXEMHFWFHG MLR LFWFOH @DFH DQV DRDD  
 SHUIRUPHG WR FRPSXWH WKH VWLGH &Y&E QVHQRVLOWV D FVRELV  
 SURJUDPV ZHUH GHYHORSHG LQ WKHMRI6Q @ VZRWKZ DVXU IWRHVEB  
 WLS VLQJXODU HOHPHQW LV FRQVWUXFW GMRLSHU LPSHQWLPD @  
 SUHVHQWHG DQG FRPSDUHG ZLWK WKHFRZDOWK M B O X G W W \  
 DQDO\VLV DSSURDFK IRU IDWLJXH OMLRH QVWLPDWHFLRQFB O O H O V  
 FRQFOXVLRQV FDQ EH GUDZQ EDVHG RQ WKH ILQGLQJV RI WKL  
 %DVHG RQ H[SHULPHQWDO GDWD RN KRW ZWKR WD VFRPISXWHD  
 PRGHO IRU FDOFXODWLQJ WKH VWLGHV MRW @ V L W D M DEF  
 HVWDEOLVKHG  
 ,Q RUGHU WR REWDLQ FRQYHUUJH B GURQXV W H L Q W H U L Q W H V  
 WKH VWHHO WXEH DQG WKH H[WHUQ @ & V X U I D F H L Q W W K H  
 EH VHSUDUWHG HYHQ WKRXJK WKH \ B O H H P M Q W K H Z B R H H  
 EHWZHHQ WKH LQWHUIDFH QRGHVHMRI FWL PXODWLRQ IULFV  
 \$ IDWLJXH OLIH HVWLPDWHVIRPQR SHUR F B G X M R L Q H G / & 7 & H IDW  
 OLIH HVWLPDWHG IURP ILQLWH HOHPHQW ZLQWKOWKLV H[SHU  
 UHVXOWV 7KH HIIHFW RI WKH LQHLVOLD @ ZDQ FNV X G J H B Q 7 W  
 LQ IDWLJXH OLIH RI ZHOGHG &)&+607GMRLQW VLQLW KDQRUP  
 IDOOLQJ ZLWKLQ WKH UDQJH VSHFLILHG E\ +6( ZDV ZLW  
 5HODWLRQVKLS EHWZHHQ WKH FUDFN MLJHWK @ G Z @ @ @ G F \&  
 7 MRLQWV 7KH PRVW SDUW RI WKMHG MR L Q W W K H D W K L D Q @  
 VWDJH ,Q VKDOORZ FUDFN VWDJH H G M S W K H G L W D H F N U R G  
 SURSDJDWHV PRUH UDSLGO\ DORQJ L W M D F @ H G J W @ O I : K Z D Q  
 WKLFNQHV FUDFN H W R W W @ L Q V E R X D F E G H L S D H F W D R Q V L R Q K  
 H[WHUQDO UHLQIRUFLQJ QHHG LWRE @ H Q V D N U H W K @ K K R  
 UHJLRQ RI WKH &)&+6 VWUXFWXUH VXEMHFWHG WR F\FOL  
 7KH ILOOHG LQ FRQFUHWH FDQ HIRFWIDYHLOXHQF UIHHD R H D  
 MRLQW 7KLV EHKDYLRU EHQHILWQF UHRAPHVZKHL FKI IHF SW RY  
 ULJLGLW\ GLVWULEXWLRQ RI WKH ZKR @ R W M R L Q W U D I Q G V  
 FRQFHQWUDWLRQ DW WKH ZHOG W R G H R R U W K E @ D H W H K R U W  
 LWVHOI KDV QR VLJQLILFDQW LQIOXHQFH RQ VWUHVV FRQ

\$&.12:/'\*(0(176

7KH DXWKRU V ZRXOG OLNH WR DFN PQW ZGHIGJMXW B B BLFQ B HQFHD Q  
RI &KLQD WKURXJK WKH JUDQWV 12 K DXW IDUG LV DOVR 7 KH DW  
.ZDQJ +XD 6FKRODU )RXQGDWLRQ RI D B Q L M L B Q D O H U Q D W F L B O  
WKL V UHVHDFK SURMHFW

127\$7,21

a 'HSWK RI D VHPL HOOLSWLFDO FUDFN  
a 'HSWK RI D LQLWLDO FUDFN  
a) 'HSWK RI D ILQDO FUDFN  
c +DOI OHQJWK RI D VHPL HOOLSWLFDO FU  
L /HQJWK RI D FKRUG  
L /HQJWK RI D EUDFH  
d ([WHUQDO GLDPHWHU RI D FKRUG  
d ([WHUQDO GLDPHWHU RI D EUDFH  
t :DOO WKLFNQHV V RI D FKRUG  
t :DOO WKLFNQHV V RI D EUDFH  
a &KRUG OHQJWK ISDUDPHWHU  
β 'LDPHWHU UDWLR EHWZHHQ D EUDFH DQG D FKRUG  
γ +DOI GLDPHWHU WR WKLFNQHV UDWLR RI D FKRUG  
τ :DOO WKLFNQHV UDWLR EHWZHHQ D EUDFH DQG D FKRUG  
hr +HLJKW RI ZHOG WRH RQ WKH FKRUG VLGH  
hr +HLJKW RI ZHOG WRH RQ WKH EUDFH VLGH  
c 2XWHU FLUFXPIHUHQFH RI D FKRUG  
c ,QQHU FLUFXPIHUHQFH RI D FKRUG  
c 2XWHU FLUFXPIHUHQFH RI D EUDFH  
c ,QQHU FLUFXPIHUHQFH RI D EUDFH  
c 3URMHFWHG OHQJWK LQ SODQH RI ZHOG WRH RQ WKH FK  
K 6WUHVV LQWHQVLY\ IDFWRU  
K, 6WUHVV LQWHQVLY\ IDFWRU RI ORGH ,  
K,, 6WUHVV LQWHQVLY\ IDFWRU RI ORGH ,,  
K,,, 6WUHVV LQWHQVLY\ IDFWRU RI ORGH ,,,  
K 6WUHVV LQWHQVLY\ IDFWRU UDQJH  
KHII (IIFWLYH VWUHVV LQWHQVLY\ IDFWRU  
γ θ 3RODU FR RUGLQDWH V\ VWHP IRU D FUDFN WLS  
\* (ODVWLF VKHDU PRGXOXV  
v 3RLVVRQ V UDWLR  
u v w /RFDO UDGLDO QrupDO DQG WDQJHQWLDO GLVSODFHPH  
& 3DULV V FUDFN JURZWK ODZ FRHIILFLHQW  
P 3DULV V FUDFN JURZWK ODZ H[SRQHQW  
N 1XPEHU RI F\FOHV  
σ QRP 1RPLQDO VWUHVV LQ D SODWH  
Q 6KDSH IDFWRU LQ 1HZPDQ 5DMX V VROXWLRQ  
6&) 6WUHVV FRQFHQWUDWLRQ IDFWRU

5()(5(1&(6

- > @ :DUGHQLHU - 3DFNHU - \$ =KDR ;+ROORZ 6BQWGRQ  
6WUXFWXUDO \$SSOLFDWLRQV' 'HOIW %RXZHQ PHW 6WDDO
- > @ =KDR ; / +DQ / + DQG /X U 0H8PE@FUWHV@G) L&RQ@GH FV  
/RQGRQ 7D\ORU )UDQFLV SS
- > @ =KDR ; / +HULRQ 6 3DFNHU \*- \$:D3&W@DHU 5-6 :6HOG  
YDQ :LQJHUGH \$ 0 DQG <HRPDQV 1U)DQGH5/HF@D@U@@DBU+  
6HFWLRQ :HOGHG -RLQWV XQGHU )DWLJXH /SSGLQJ' &,'(&7
- > @ /HH & &KLHZ 6 /LH 6 DQG 6TXDUH)DWL6TXHD@H K+RYL  
7 MRLQW ZLWK &RXP@HUL F@D@D @R GH,O L@U D F @XW @ H@HUF K@D@L F V  
9RO 1R SS
- > @ /HH & . &KLHZ 6 3 DQG /OH6V6XG\ HR@ B@U H(VSH&R@F  
)DFWRUV IRU 3DUWLDOO\ 2YHUODSSHV' &L\$GX DQ@B@H@R@O@F  
&RQVWUXFWLRQ 9RO 1R SS
- > @ YDQ :LQJHUGH \$ 0 3DFNHU)D-WLJ@G@XDU@GH@U@H@X@E@O@  
,,: ,QWHUQDWLRQDO &RQIHUHQFH RQ H@G@U@H@O@G@D@G@F@H@V@U@X@Q@F@W@X@  
6DQ )UDQFLVFR 86\$ SS
- > @ 7RQJ / DQG ; / =KDR HW DG 7KDW LZXDH@H@B@K@D@M@R@L@Q@W@  
&LUFXODU DQG 6TXDUH +ROORZ 6H F@M@H@O@G@W@U@Q@F@W@X@U@H@D@W@L@R@G@  
1R SS
- > @ 8GRPZRUDUDW 3 0LNL & P,FWRNDZD@D@S@W@V@K@D@L@D@N@L@D@Q@G@  
7 3)DWLJXH DQG B@O@V@R@I@P@O@R@C@F@G@M@W@H@Q@M@V@O@H@G@V@7@R@E@X@D@U@X@V@V@\*@L@  
-RXUQDO RI 6WUXFWXUDO (QJLQHHULQJ 9RO \$ SS
- > @ 7RQJ / : 6XQ & 4 &KHQ <QG AKDR& %/ 3@KSH@L@P@H@  
&RPSDULVRQ LQ +RW 6SRW 6WUHV@V@E@W@W@Z@D@B@Q@&@J@R@F@H@D@G@G@Q@J@  
WK ,QWHUQDWLRQDO 6\PSRVLXP R@Q@Q@E@X@O@D@U@S@S@W@U@X@F@W@X@U@H@  
> @ 0DVKLUL ) 5 DQG =KDR ; / L@G@W@X@D@Z@U@H@W@K@R@O@R@Z@U@G@H@F@H@L@R@  
VXEMHFWHG WR LQ SODQH )DWLJXH@G@/R@D@G@K@Q@W@X@L@Q@H@W@K@H@%@U@D@F@  
1R SS
- > @ =KDR ; / DQG 7RQJ / : 31HZ 'O@D@H@O@R@S@D@H@Q@W@L@S@G@Y@D@Q@F@  
6WUXFWXUDO (QJLQHHULQJ 9RO 1R SS
- > @ :DQJ . 7RQJ / : =KX - @K@=K@R@Q@D@V@K@E@)D@L@U@J@%@H@D@%@H@  
RI :HOGHG 7 MRLQWV EHWZHHQ &+6 %U@D@F@H@R@D@G@G@&)&@G@6@W@K@  
%UDFH' -RXUQDO RI %ULGJH (QJLQHHULQJ 9RO 1R
- > @ +REEDFKHU \$ 36WUHV@V@,QWH(Q@V@L@Q@H@H@D@E@V@R@U@V@D@F@I@W@X@I@D@G@H@  
9RO 1R SS
- > @ %RZQHVV ' DQG /HH 0 0 . 37K@H@H@Y@R@G@R@S@P@H@Q@W@W@K@H@D@Q@  
\$V@V@H@V@V@P@H@Q@W@RI'R@X@E@O@&X@U@Y@H@G@&U@L@R@N@D@Q@Q@F@X@I@E@Q@D@D@U@R@F@R@U@D@  
9RO 1R SS
- > @ \$PHULFDQ :HOG@L@Q@J@6R@F@L@H@W@\ \$@V@H@H@3@G@W@U@W@F@K@W@X@G@U@H@D@H@D@L@Q@  
SS
- > @ +HDOWK DQG 6DIHW@ ([HFXWLY@H@J@X@+6(\*X@L@3@G@D@F@H@U@R@X@Q@G@W@W@R@  
DQG &RQ@Q@H@F@W@L@R@Q@V@L@Q@2@I@I@V@K@R@U@H@6W@U@X@F@W@X@U@H@V' 26S7HFK
- > @ \$16<6 5HOHDVH +HOS 6\V@W@H@E@G@H@&R@X@S@O@H@G@,Q@F@H@O@G@S@Q@D@
- > @ %DUVRXP 5 6 37ULDQJXODU@W@L@X@D@D@Q@G@H@S@R@H@Q@W@Q@O@B@B@Q@W@  
(O@H@P@H@Q@W@V' ,QW - 1XP 0HWK (QJ 9RO 1R SS

- > @ &KLHZ 6 3 /LH 6 7 /HH &V.V , Q V G Q X L D W Q J ) D F W R U 6 / W I U R I  
&UDFN LQ D 7XEXODU 7 MRLQW' 9 Q W W H Q Q V D I Q R Q B I O S E R X I U Q D  
1R SS
- > @ %RZQHVV ' /HH 0 0 . 3\$ ) L Q V W ) H L H O G P H Q Q G 6 W X L G H V R I  
)DFWRUV LQ 7XEXODU -RLQWV' - R X U Q D O I R I 6 V S D L Q \$ Q D O \
- > @ 1HZPDQ - & DQG 5DMX , 6 W 3 \$ Q D F W S L W L F D O V 6 L W R L Q H V R U , O  
&UDFN' (QJLQHHULQJ)UDFWXUH 0 H S K D Q L F V 9 R O
- > @ %ULWLVK 6WDQGDUGV , Q V W L W X V S V R Q V 6 L Q J W P K X L \$ F H F R Q V  
)ODZV LQ 0HWDOOLF 6WUXFWXUHV' /RQGRQ 8. SS
- > @ 3DULV 3 & 3 & U G W U F D Q \$ Q D O S L M D V L R Q D D V 3 U \$ 6 0 ( - R X U  
%DVLV (QJLQHHULQJ 9RO 1R SS
- > @ %ULWLVK 6WDQGDUGV , Q V W L W X I R U R \$ V 8 H V V L Q \* X W K D Q \$ F H F R  
R I ) O D Z V L Q ) X V L R Q : H O G H G 6 W U X F W X U H V ' / R Q G R Q 8 .
- > @ 6FKXPDKHU \$ DQG 1XVVEDXPHU \$ H 3 [ S M U L P M Q W D O  
:HOGHG 7XEXODU . MRLQWV IRU %ULG 9 R V ' ( Q J L Q H H U L Q J 6 V
- > @ %RUJHV / DQG 1XVVEDXPHU \$ J 3 \$ I G Y D V Q L F J H G H 1 6 X P H U L I F D F  
:HOGHG &+6 . MRLQWV' 3URFHHGLQXP R R Q 7 X E X Q D U H 6 Q D W E I  
&KLQD SS
- > @ 1XVVEDXPHU \$ DQG +DOGLPDGQH 6 W X Q W V 6 X & L Q 3 J ) D M Q G X H  
RU &DVW 6WHHO 1RGH 6ROXWLRQLR Q D R F H P S R Q L X P R R Q V K M  
6WUXFWXUHV /RQGRQ SS