DESIGN OF PLANE STEEL FRAMES – TOWARDS A RATIONAL APPROACH

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Abstract: Some concepts and results dealing with the design and safety checking of members integrated in plane steel frames are presented and discussed. Initially, attention is paid to the identification and clarification of a number of ambiguities related to the application of the buckling length concept to frame members. Then, the safety checking of columns integrated in frames is addressed and it is shown that, if their buckling lengths are "correctly determined", only one particular column, designated as "critical column", needs to be checked – a finding which leads to the proposal of a "frame optimisation procedure". Next, the safety checking of beam-columns integrated in frames is dealt with: the application of the interaction formulae appearing in the upcoming EN version of Eurocode 3 is addressed and particular attention is paid to the appropriate choice of the buckling length and "equivalent moment factor" values, both in terms of safety and accuracy. In addition, one proposes an alternative approach to use the beam-column interaction formulae, which is based on the results of genuine second-order elastic analyses. In order to illustrate the application and assess the validity and advantages of the concepts and procedures presented throughout the paper, one presents numerical results concerning simple (two-bar) structural systems and these results are compared with "exact" frame ultimate (collapse) load values, yielded by second-order plastic zone analyses that incorporate member initial imperfections. On the basis of the above comparative study, it is possible to draw several conclusions and, in particular, it is shown that the proposed approaches consistently yield accurate and conservative frame strength estimates.

Key words: Steel frames, columns, beam-columns, buckling length, equivalent moment factors, non-dimensional slenderness, frame slenderness, Eurocode 3.

1. INTRODUCTION

Ideally, the design or safety checking of steel frames should be carried out on the basis of rigorous geometrically non-linear elastic-plastic structural analyses, which incorporate member and frame imperfections and provide "exact" frame load-carrying capacities. However, in spite of the fast growing popularity of the so-called "advanced methods of structural analysis"[1,2] – some of them are already allowed by various existing steel design codes (*e.g.*, the current and upcoming versions of Eurocode 3 or simply EC3 [3,4]) –, their use remains prohibitive for routine applications. This stems mostly from the fact that (i) the computational effort required is still quite high and (ii) the vast majority of designers lack the appropriate theoretical background. Thus, the most "traditional" approach, based on first-order internal forces and moments and individual member checks through beam-column interaction formulae, continues to be widely adopted by practitioners. Nevertheless, since the interaction formulae must incorporate all relevant second-order and plasticity effects, an intense research activity is still going on concerning the improvement of such formulae, which aims at making them as accurate, rational, general and easy-to-use as possible.

As far as the design of frame compressed members is concerned, it is common practice to employ "column buckling curves", an approach that requires the adoption of appropriately chosen elastic buckling lengths (L_{cr}), in order to adequately simulate the behaviour of the member *within the frame under consideration*. Therefore, since the "classical" elastic buckling length concept still plays a very relevant role in the frame design and/or safety checking procedure, it is very important to define it properly and also to provide the means to evaluate its correct value.

In the particular case of EC3, the beam-column interaction formulae appearing in both the available (European Pre-Norm − ENV [3]) and upcoming (European Norm − prEN [4][1]) versions were developed, calibrated and validated (experimentally and/or numerically) almost exclusively in the context of isolated and simply supported members (*e.g.*, the so-called "European buckling curves" [5,6]). Investigations aimed at assessing the efficiency (accuracy and safety) of the application of these formulae to members integrated in frames, are still scarce. This stems mostly from the fact that codes invariably assume (implicitly) that the frame behaviour can be adequately simulated through the choice of appropriate (i) buckling lengths and (ii) equivalent moment factors (C_m). However, it seems fair to say that the specific issues related to the actual choice of such buckling lengths and equivalent moment factors can seldom be found in the literature.

This paper addresses issues related to the application of the EC3 buckling formulae to compressed members (columns and beam-columns) integrated in plane frames. The chief objective of this work is to present and discuss results that will (i) contribute to a better understanding of the formulae fundamentals and (ii) pave the way to the establishment of guidelines to use them more efficiently. It is worth noting that only the member in-plane behaviour associated with major axis bending is dealt with and, moreover, that particular emphasis is given to the importance, in terms of both safety and accuracy, of appropriately choosing the values of L_{cr} and C_m.

Initially, one tackles the safety checking of columns (uniformly compressed members) integrated in frames. After a brief review of the EC3 buckling provisions, attention is paid to the identification and clarification of a number of ambiguities that are commonly associated with the application of the buckling length concept to frame members. Then, one shows that, provided that the buckling lengths are "correctly determined", only the safety of one particular column − designated henceforth as "critical column" − needs to be checked. This finding provides valuable insight concerning the frame overall behaviour and, in particular, makes it possible (and fairly easy) (i) to identify and/or strengthen the frame "weaker" members and also (ii) to develop a frame optimisation procedure.

Next, the design and safety checking of beam-columns (members subjected to compression and bending) integrated in frames is dealt with, namely by discussing the so-called *Method 1* and *Method 2* beam-column interaction formulae appearing in EC3-prEN [4]. Besides addressing issues concerning the choice of the appropriate L_{cr} and C_m values, one also proposes an alternative approach to the use of the above formulae, which involves the use of internal force and moment values that are obtained from genuine second-order elastic analyses of "ideal" (initially "perfect") frames − this approach does not require the incorporation of initial imperfections in the frame analysis.

Finally, it is still worth mentioning that the concepts and procedures presented throughout this paper are illustrated by means of their application to simple (two-bar) structural systems. Moreover, in order to assess the validity of these concepts and procedures, one compares the member strength estimates yielded by them with "exact" results, *i.e.*, results obtained from second-order elastic-plastic (plastic zone) finite element analyses, which (i) incorporate standard initial geometrical imperfections and

[1] At this moment, there is only a "preliminary" (but practically "final") version of the upcoming EC3 document.

residual stresses and (ii) are performed using the commercial code ABAQUS [7]. The results of this (obviously limited) comparative study are very promising, in the sense that they provide clear evidence that the proposed approaches and methodologies consistently yield accurate and conservative strength estimates. On the basis of the above findings, it is possible to draw several important conclusions and to anticipate the formulation of rather general guidelines for the rational design and safety checking of plane steel frames – to be subsequently validated, of course.

2. DESIGN AND SAFETY CHECKING OF COLUMNS INTEGRATED IN FRAMES

2.1 Buckling Resistance According to Eurocode 3

According to EC3, the safety of columns with class 1, 2 or 3 cross-sections is ensured by the condition

$$n_{b,Rd} = \frac{N_{Ed}}{N_{b,Rd}} = \frac{N_{Ed}}{\chi N_{pl}/\gamma_{M1}} \leq 1 \qquad (1)$$

where N_{Ed} is the acting compressive force design value, $N_{b,Rd}$ is the flexural buckling resistance, $N_{pl}=Af_y$ is the plastic axial force (A is the cross-section area and f_y is the characteristic yield stress), γ_{M1} is the partial resistance factor for member instability and χ is the reduction factor for flexural buckling, which takes into account the influence of the geometrical and material non-linear effects and member imperfections. The value of χ is obtained from the appropriate buckling curve, which is defined by the expressions

$$\chi = \frac{\Psi}{2\bar{\lambda}^2} \quad \psi = \min(2\bar{\lambda}^2; 1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2 - \sqrt{(1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2)^2 - 4\bar{\lambda}^2}) \qquad (2)$$

where α is an imperfection factor, the value of which characterises each buckling curve, and

$$\bar{\lambda} = \sqrt{N_{pl}/N_{cr}} \qquad (3)$$

is the column normalised, (non-dimensional) slenderness – $N_{cr}=\pi^2 EI/L_{cr}^2$ is the elastic critical axial force, L_{cr} is the column critical *buckling length*, I is the cross section second moment of area about the relevant axis and E is Young's modulus.

It is important to draw the reader's attention to the fact that the key step in the safety checking procedure defined by Eq. (1) is the evaluation of the critical buckling length L_{cr} (or, equivalently, of N_{cr}). In fact, once L_{cr} is known, the calculation of all the other parameters involved is extremely easy and quite straightforward. However, it will be shown in the next section that the application of the buckling length concept to members integrated in frames can lead to some rather curious and unexpected results – in fact, this concept is only well established and perfectly unambiguous for isolated uniform members [8,9].

2.2 Buckling Length of Columns Integrated in Frames

Although the buckling length concept has been extensively covered in the structural stability literature (*e.g.*, [8,9]), its application to members integrated in frames is by no means obvious – it still raises novel issues and poses interpretation problems [10,11]. In fact, it is fair to say that the difficulties associated with the proper use of the buckling length concept in frame members have led to a current trend that advocates either (i) its elimination from the design rules (*e.g.*, [12-16]) and/or (ii) its replacement by global frame parameters and "frame stability curves" [17,18].

The (critical) buckling length L_{cr} of a uniformly compressed bar with constant cross-section is commonly defined as:

(i) The length of a fictitious isolated and simply supported, but otherwise identical, column that buckles for the same axial force value $N_{cr}=\pi^2 EI/L_{cr}^2$.

(ii) The distance between two consecutive inflection points (*i.e.*, points of zero moment) of the corresponding buckling mode $\bar{w}$, which is given by [19]

$$\bar{w}(x) = A_1 \sin\left(\frac{\pi x}{L_{cr}}\right) + A_2 \cos\left(\frac{\pi x}{L_{cr}}\right) + A_3 x + A_4 \qquad (4)$$

where x is a coordinate along the member axis and A_1–A_4 are constants depending on the column boundary conditions. By differentiating Eq. (4) twice with respect to x, one obtains a sinusoidal function with a period equal to $2L_{cr}$, thus proving that the distance between two consecutive inflection points is half the period, *i.e.*, exactly L_{cr}.

Since the above definitions apply for columns integrated in frames, a rigorous evaluation of their buckling lengths requires the performance of a linear stability analysis of the whole frame [20], which (i) must account for the axial forces acting on *all* the columns and (ii) provides the frame critical load parameter value Λ_{cr}. Then, the critical axial force for each column is given by

$$N_{cr,i} = \Lambda_{cr} \bar{N}_i \qquad (5)$$

where $\bar{N}_i$ is the corresponding reference axial force (*i.e.*, associated with $\Lambda=1$), and leads to the determination of its buckling length, which is obtained from the expression

$$L_{cr,i} = \sqrt{\frac{\pi^2 E_i I_i}{N_{cr,i}}} = \sqrt{\frac{\pi^2 E_i I_i}{\Lambda_{cr} \bar{N}_i}} \qquad (6)$$

On the basis of Eq. (6) it is possible to establish the relation between the L_{cr} values of two columns belonging to the same frame (for a particular load combination), which reads

$$\frac{L_{cr,i}}{L_{cr,j}} = \sqrt{\frac{E_i I_i}{E_j I_j} \frac{\bar{N}_j}{\bar{N}_i}} \qquad (7)$$

and shows that the buckling lengths of all the frame columns are related to each other through their (i) flexural stiffness values EI and (ii) reference axial forces $\bar{N}$. This particular feature reveals that frame members with large EI and/or small $\bar{N}$ values tend to exhibit rather high L_{cr} values. For illustrative purposes, consider the inverted L-frame depicted in Figure 1(a), subjected to two compressive loads P_1 and P_2. Figures 1(b) and 1(c) display the K-factors ($K_i=L_{cr,i}/L_i$) associated with each compressed member, obtained from exact linear stability analyses, for several load and bending stiffness combinations. One notices that, as predicted by Eqs. (6) and (7), decreases in P_2 and $(EI)_2$ lead to higher K_2 and K_1 values, respectively.

This example clearly illustrates that the application of the buckling length concept to members integrated in frames may lead to unexpected results. Note also that, although the frame is braced (and, thus, classified as "non-sway"), the column K-factor can be (much) higher than 1. However, it must be stressed that these results stem from the fact that one is somewhat "mixing" the buckling behaviours of frames and isolated columns. For instance, Figure 1(c) shows that an $(EI)_1$ increase leads to a growth in both $N_{cr,1}$ (obviously, because the flexural restraint is higher for column 2 and one has $P_1=P_2$) and K_1 – the $(EI)_1$ increase is much more relevant that the $N_{cr,1}$ one.

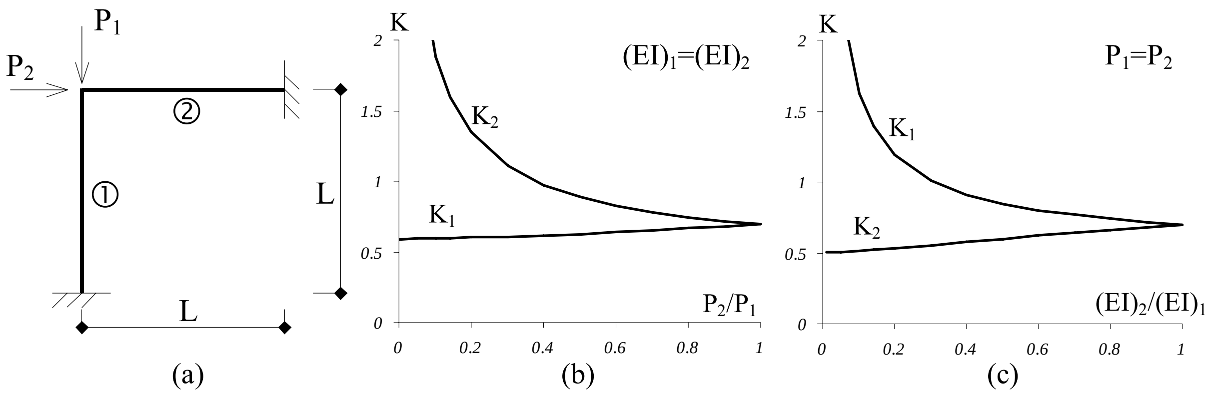

Figure 1. Inverted L-frame (a) geometry/loading and K-factors for (b) $(EI)_1=(EI)_2$ and (c) $P_1=P_2$

These examples clearly illustrate that the application of the buckling length concept to frame members may lead to unexpected results. Notice that, although the frame is obviously classified as "non-sway", the K-factors can be much higher than 1. However, it should be highlighted that these results stem from the fact that one is trying to determine "the length of a fictitious isolated and simply supported (but otherwise identical) column that buckles at the axial force level defined by the frame instability" – for instance, if $P_1=P_2$ and $(EI)_2$ is kept constant, it is obvious that $N_{cr,1}$ increases with $(EI)_1$, although one also observes a K_1 increase (see Figure 1(c)).

Moreover, from Eqs. (3) and (5), one readily concludes that the relation between the normalised slenderness values of columns i and j is given by

$$\frac{\bar{\lambda}_i}{\bar{\lambda}_j} = \sqrt{\frac{N_{pl,i}}{N_{cr,i}}\frac{N_{cr,i}}{N_{pl,j}}} = \sqrt{\frac{N_{pl,i}}{N_{pl,j}}\frac{\bar{N}_j}{\bar{N}_i}} \tag{8}$$

which shows that the $\bar{\lambda}$ values of all the frame columns are related to each other through their relative N_{pl} and $\bar{N}$ values. This leads to a quite curious result, namely that the column that is closer to reaching N_{pl} is the most "slender" one, *i.e.*, has the highest $\bar{\lambda}$ value. This result has the following implications:

(i) Among the members exhibiting the same N_{pl} value, the one acted by the lowest axial force N_{Ed} is the most "slender".

(ii) Among the members acted by the same axial force, the one exhibiting the highest N_{pl} value is the most "slender".

Since high $\bar{\lambda}$ values imply low $N_{b,Rd}$ values, such members are often regarded as "weak", in the sense of "highly susceptible to second-order effects", although this perception seems to disagree with the fact that they may be subjected to low N_{Ed} values or display high N_{pl} values. Although these (apparently) paradoxical conclusions are addressed and explained later in the paper, it is important to emphasise right away that, in the case of frame members, a high $\bar{\lambda}$ value does not necessarily imply a "low buckling resistance" or " high susceptibility to second-order effects" – it merely indicates that the member critical axial force is, in the specific context of the loaded frame under consideration, well below the associated plastic axial force (recall Eq. (3)). It follows that, in order to adequately assess the susceptibility of a given frame to second-order effects, it is necessary to define a "global" parameter. In the next section, such parameter, the "frame slenderness", is presented and discussed.

2.3 The Concept of Frame Slenderness

The concept of "normalised (non-dimensional) slenderness", traditionally used in the context of compressed structural elements (columns, beams, plates, etc.), can be readily extended to frames, by resorting to the definition [18,21][2]

$$\bar{\lambda}_f = \sqrt{\Lambda_y/\Lambda_{cr}} \tag{9}$$

where Λ_{cr} is the frame critical load parameter and Λ_y is the frame "yielding" load parameter, which is associated with the yielding of the first compressed member in the whole frame, due to axial force alone (*i.e.*, the one first reaching its plastic axial load), and is defined by

$$\Lambda y = \min\left(\frac{A_i f_{y,i}}{N_i}\right)(i = 1,...,n) \tag{10}$$

where n is the number of frame compressed members[3]. Both Λ_{cr} and Λ_y must be related to the same load combination and, for an "ideal frame" (*i.e.*, geometrically/materially perfect and with no first or second-order bending moments acting on its members), $\bar{\lambda}_f > 1\,(<1)$ indicates that frame buckling precedes (follows) the attainment of the plastic load in any member. Then, one may express the load-carrying capacity or "resistance" of an "ideal frame" (Λ_R) in terms of the *frame reduction factor* χ_f

$$\Lambda_R = \chi_f \Lambda_y \tag{11}$$

$$\chi_f = \frac{\min(\Lambda_y \Lambda_{cr})}{\Lambda_y} = \min\left(1; \frac{1}{\bar{\lambda}_f^2}\right) \tag{12}$$

It is worth noting that the variation of χ_f with $\bar{\lambda}_f$ is analogous to its "ideal column" counterpart (Eq. (2), making α=0). By incorporating (5) and (10) into (9), one concludes that

$$\bar{\lambda}_f = \min(\bar{\lambda}_i) = \bar{\lambda}_{min}\ (i = 1,...,n) \tag{13}$$

i.e., that $\bar{\lambda}_f$ is precisely the smallest among the $\bar{\lambda}$ values of all frame columns.

A nice feature of the frame slenderness concept resides in the fact that, like for isolated columns, the designation "slender frame" means "frame susceptible to second-order effects", thus making it possible to circumvent the problems associated with the interpretation of the buckling lengths and normalised slenderness values of each individual column, which was explained in subsection 2.2. Moreover, the frame slenderness concept deserves a few important remarks:

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[2] It is worth noting that this concept also appears in the EC3-prEN [4], defined as "relative slenderness of the structure".

[3] Although the tensile members can be taken into account when determining Λ_{cr}, they must not be involved in the calculation of Λ_y – the safety checking of the tensile members has to be performed separately and independently.

(i) Like for isolated members (*e.g.*, columns, beam-columns), the calculation of $\bar{\lambda}_f$ does not take into account the various member bending moments and shear forces. Moreover, it is theoretically possible to obtain *any* $\bar{\lambda}_f$ value associated with a given Λ_{cr}, by merely changing the plastic axial forces (N_{pl}) of the frame members, *i.e.*, changing Λ_y.
(ii) Since, in most situations, the first and second-order member axial force values are almost identical, Λ_y can be determined from Eq. (10), which is based on first-order axial forces.
(iii) It might be argued that, since Λ_y underestimates the frame plastic load-carrying capacity, a "plastic load parameter" Λ_{pl}, corresponding to the formation of a complete frame collapse mechanism, could be used instead. However, because the determination of such parameter is not at all easy and the main goal behind the use of $\bar{\lambda}_f$ is just to assess the susceptibility of the frame to second-order order effects, one adopts only Λ_y in this work.
(iv) It is important to check whether the compressed members that yield for $\Lambda=\Lambda_y$ also participate in the frame critical buckling mode (associated with $\Lambda=\Lambda_{cr}$). This ensures that Λ_{cr} and Λ_y concern the same structural system or sub-system.

2.4 Column Design and Safety Checking

The design or safety checking of columns integrated in a frame is performed by means of Eq. (1). Using Eqs. (2), (3), (5) and introducing the design load parameter $\Lambda_{Ed} = N_{Ed,i}/\bar{N}_i$, any frame column i must satisfy the condition

$$n_{b,Rd,i} = \frac{N_{Ed,i}}{N_{b,Rd,i}} = \frac{\Lambda_{Ed}\bar{N}_i}{\frac{\chi_i N_{p1,i}}{\gamma_{M1}}} = \frac{2_{\gamma M1}}{\Psi_i}\frac{\Lambda_{Ed}}{\Lambda_{cr}} = \frac{2_{\gamma M1}}{\Psi_i} n_{cr} \leq 1\,(i = 1,...,n) \tag{14}$$

where $n_{cr}=\Lambda_{Ed}/\Lambda_{cr}$ and it is important to realise that the only parameter that changes from column to column is Ψ_i. The curves depicted in Figure 2 provide the variation of Ψ with $\bar{\lambda}$, for (i) an "ideal" column and (ii) the EC3 column buckling curves *a–d* [3,4]. Because Ψ grows monotonically with $\bar{\lambda}$ in all these curves, one readily concludes from Eq. (14) that *the column with the lowest* $\bar{\lambda}$ ($=\bar{\lambda}_{min}$) *is the one governing the safety checking of the whole frame* – such column will be hereafter designated as "critical" and denoted by $(\cdot)_c$[4]. Moreover, from Eq. (13) one also deduces that *the critical column* $\bar{\lambda}$ *value* ($\bar{\lambda}_c$) *is equal to the frame slenderness value* (*i.e.*, $\bar{\lambda}_c=\bar{\lambda}_{min}=\bar{\lambda}_f$). Although, strictly speaking, the above conclusions are only one hundred per cent true whenever the same buckling curve applies to all columns, it seems quite safe to say that, when performing the design or safety checking of a given frame, columns with $\bar{\lambda} >> \bar{\lambda}_f$ do not need to be checked – they do not govern the frame safety checking and should be viewed as "over designed", as far as strength is concerned.

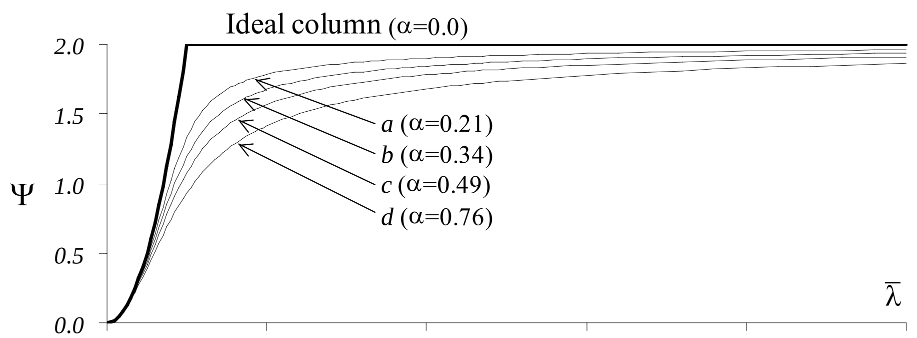


Figure 2. Variation of Ψ with $\bar{\lambda}$ for (i) an "ideal" column and (ii) the EC3 curves *a–d*

[4] Obviously, a frame may have several critical columns, all of them sharing $\bar{\lambda}_{min}$.

On the other hand, since the critical column always exhibits the lowest $\bar{\lambda}$ value (if the same buckling curve applies for all columns), it is possible to conclude, on the basis of Eqs. (5), (8), (13), that this critical column is the one *closest to reaching the plastic axial force* $N_{pl}=Af_y$. Indeed, one has

$$\bar{\lambda}_c = \lambda_{min} = \min(\lambda_i) = \sqrt{\min\left(\frac{A_i f_{y,i}}{\Lambda_{cr}\bar{N}_i}\right)} = \sqrt{\frac{1}{\Lambda_{cr}}\min\left(\frac{A_i f_{y,i}}{\bar{N}_i}\right)} \ (i = 1,...,n) \qquad (15)$$

Consequently, a frame in which all columns (i) exhibit the same cross-section area and steel grade and (ii) are governed by the same buckling curve, *the critical column is always the one acted by the highest axial force, regardless of its length*.

If the columns are governed by different buckling curves (a usual case), one must consider the possibility that the critical column may have a $\bar{\lambda}$ value higher than $\bar{\lambda}_{min} = \bar{\lambda}_f$. As an example, consider a column with $\bar{\lambda} = \bar{\lambda}_{min} = 2.0$ and governed by curve *a* (Ψ=1.783). This column will not be critical if, for instance, there is another column (i) governed by curve *d* and (ii) having $\bar{\lambda} < 6.2$ – one will then get Ψ<1.783. The charts shown in Figure 3 can be used to detect the situations in which $\bar{\lambda}_c > \bar{\lambda}_{min} = \bar{\lambda}_f$: each of them plots a set of curves providing the ratios between the $\bar{\lambda}$ values of any two columns (columns i and j, such that $\bar{\lambda}_i < \bar{\lambda}_j$) that lead to *identical* Ψ values, when column i is governed by curve *a* (left chart), *b* (middle chart) or *c* (right chart) – obviously, a column governed by curve *d* is always the critical one, since it exhibits the lowest Ψ function (see Figure 2), and no chart is needed. In the previous example, column i is governed by curve *a*, which means that one must consider the left chart, and column j by curve *d*. Then, the "*d*" curve of the chart provides, for $\bar{\lambda}_i = 2.0$, $\bar{\lambda}_j / \bar{\lambda}_i = 3.10$, thus showing that column j is critical for $\bar{\lambda}_j < 3.10 \times 2.0 = 6.2$.

It is worth noting that the critical column concept provides a theoretical validation of the "frame stability curve" concept proposed by Cosenza *et al.* [18], which prescribes the use of a single buckling curve, together with $\bar{\lambda}_f$, to estimate a frame load-carrying capacity by means of Eq. (11). It is now clear that both concepts are equivalent provided that (i) $\bar{\lambda}_c = \bar{\lambda}_{min} = \bar{\lambda}_f$ and (ii) the critical column buckling curve is used to calculate χ_f.

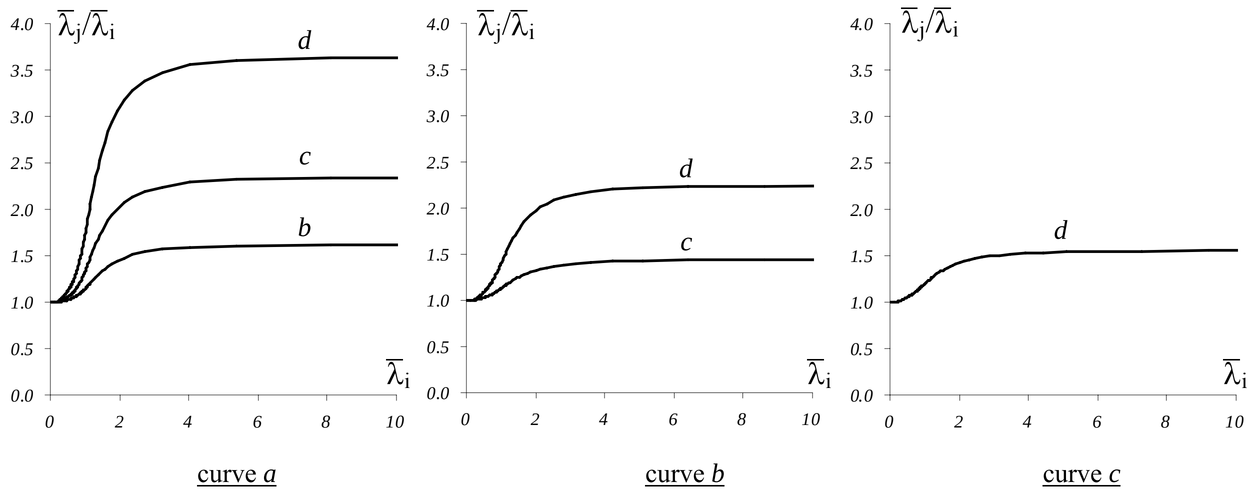


Figure 3. $\bar{\lambda}_j / \bar{\lambda}_i$ values associated with $\Psi_i = \Psi_j$ – column i corresponds to curves *a*, *b* and *c*

2.4.1 Frame optimisation procedure

On the basis of the results just presented, one readily concludes that, in an efficiently (optimally) designed frame, all compressed members must satisfy the condition (see Eq. (14))

$$n_{b,Rd,i} = \frac{2\gamma_{M1}}{\Psi_i} n_{cr} = 1 \; (i = 1,...,n) \qquad (16)$$

which means that (i) they are all critical and also that (ii) their buckling resistances are being fully used (*i.e.*, one has $N_{Ed,i}=N_{b,Rd,i}$). Such a situation can be reached by adopting the following procedure:

(i) First, at the pre-design stage, one must try to ensure that the $n_{cr}=\Lambda_{Ed}/\Lambda_{cr}$ values (one per load combination) are as low as possible − obviously, they must be lower than 1. Of course, this means that the Λ_{cr} values must be as high as possible, which can be accomplished by properly bracing the members and/or by choosing sufficiently stocky cross-sections.

(ii) For each load combination (*i.e.*, n_{cr} value), select the optimum values of $N_{pl,i} = A_i f_{y,i}$ for all compressed members, which must satisfy Eq. (16)) – note that, by changing $N_{pl,i}$, one alters $\bar{\lambda}_i$ and, thus, also Ψ_i. The chart shown in Figure 4(a) provides valuable help to perform this task, as it plots the optimum $n_{pl,i}=N_{Ed,i}/N_{pl,i}$ values as a function of n_{cr}, for an "ideal" column and the EC3 buckling curves *a–d*, adopting γ_{M1}=1.1 − a $n_{pl,i}$ value *above* (below) the curve under consideration corresponds to an *unsafe* (safe) design[5]. On the other hand, the chart displayed in Figure 4(b) provides the optimum $\bar{\lambda}_i$ values − a $\bar{\lambda}_i$ value *above* (below) a given curve is associated with *safe* (unsafe) designs.

Since the above procedure must be carried out for each load combination, it is virtually impossible to design the frame so that Eq. (16) holds for *all* members and load combinations. However, the use of the concepts on which this approach is based enables a much more rational frame design and, in addition, provides valuable insight on the frame structural behaviour – *e.g.*, the "critical" and "over-designed" compressed members can be easily spotted and, if necessary, efficiently redesigned.

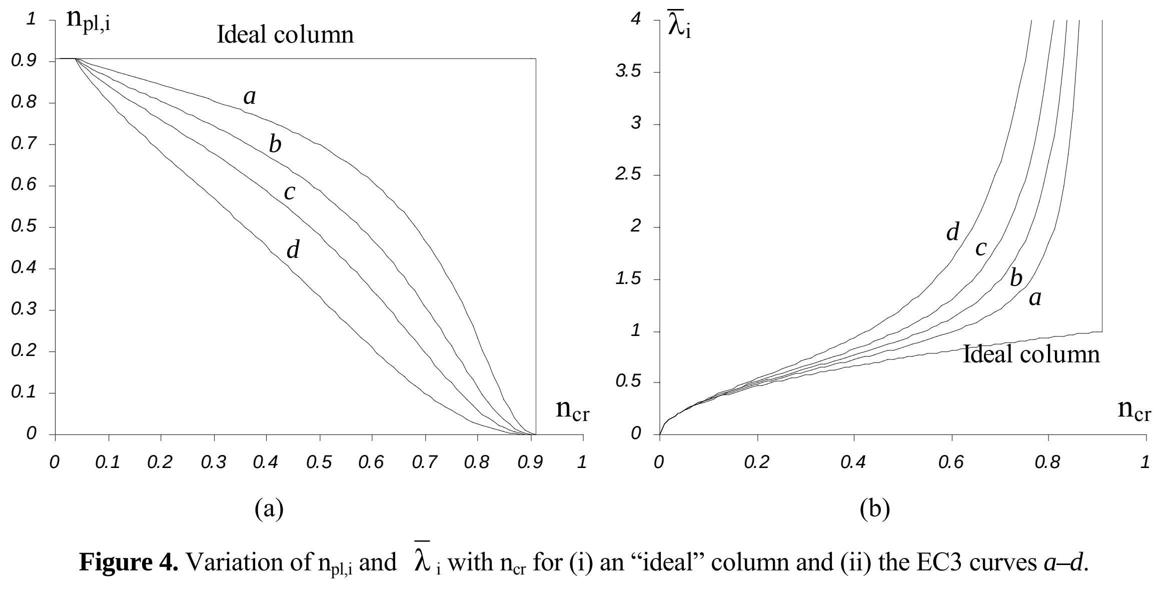


Figure 4. Variation of $n_{pl,i}$ and $\bar{\lambda}_i$ with n_{cr} for (i) an "ideal" column and (ii) the EC3 curves *a–d*.

[5] Note that a low n_{cr} value implies that the column can almost reach its plastic strength, *i.e.*, that the corresponding optimum $n_{pl,i}$ value is very close to $1/\gamma_{M1}\approx0.9$.

It is well known that, like in the case of isolated members, an efficiently designed frame exhibits members with cross-sections displaying high second moments of area I and low areas A (*i.e.*, high radii of gyration). However, due to either limitations in the range of commercially available profiles, the occurrence of local buckling phenomena or even architectural demands, it is often not possible to obtain any desired I–A combination. Therefore, the member I values are inevitably modified during the performance of the optimisation procedure – if these modifications are significant, it will be necessary to recalculate Λ_{cr} and restart the whole procedure.

For illustrative purposes, let us consider the central column of the "simple construction" frame (continuous columns and hinged beam-to-column joints) depicted in Figure5, which comprises four equal-length uniformly compressed segments. Assuming that the column has been *already* pre-designed, the current goal is to reduce the cross-section area of each segment for the load case under consideration ($\overline{N}_1$=P, $\overline{N}_2$=2P, $\overline{N}_3$=3P, $\overline{N}_4$=4P). Supposing, for the sake of simplicity, that the same buckling curve governs all column segments, the optimisation procedure involves the following steps:

(i) Performance of a linear stability analysis of the whole column, thus obtaining Λ_{cr} and n_{cr}.
(ii) Calculation of all the segment $\overline{\lambda}$ values ($\overline{\lambda}_1$–$\overline{\lambda}_4$). Since the same buckling curve applies, they are all critical if $\overline{\lambda}_1=\overline{\lambda}_2=\overline{\lambda}_3=\overline{\lambda}_4$ (=$\overline{\lambda}_f$) – from Eq. (8), this holds true if $N_{pl,2} = 2N_{pl,1}$,$N_{pl,3} = 3N_{pl,1}$ and $N_{pl,4} = 4N_{pl,1}$, regardless of their lengths and support conditions.
(iii) Calculation of the "optimum" n_{pl} value corresponding to n_{cr} for *just one (any)* segment, by using the chart in Figure 4(a) – if the N_{pl} ratios mentioned in the previous item are retained, no calculations involving the other column segments are necessary.

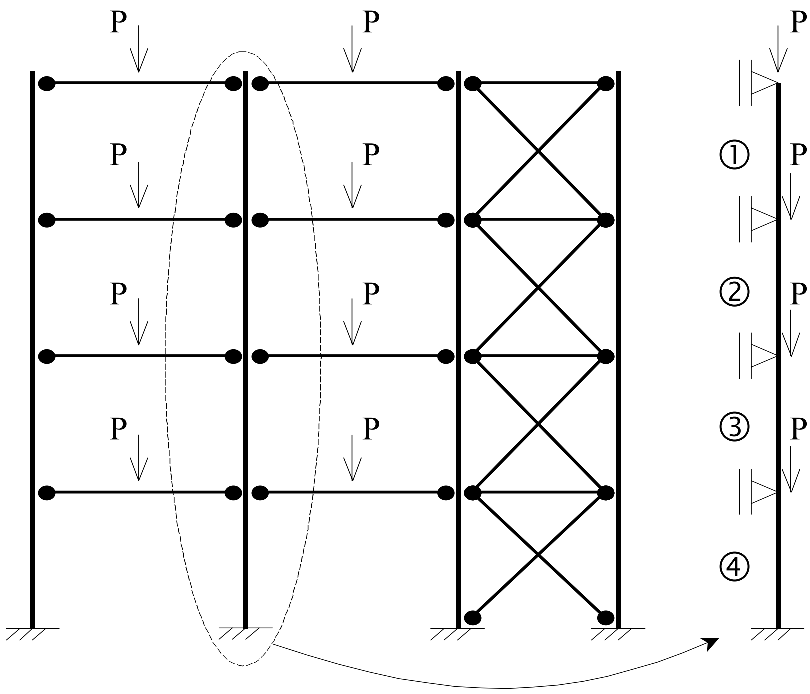


Figure 5. Illustrative example: simple construction frame

Notice, once more, that if the above procedure modifies the cross-sections I values significantly, it is indispensable to recalculate Λ_{cr} and redo the whole procedure. Moreover, if the segments are governed by different buckling curves, some extra care is required because some of the segments may not be critical for $\overline{\lambda}_1=\overline{\lambda}_2=\overline{\lambda}_3=\overline{\lambda}_4$. Finally, it is important to stress again that the conclusions drawn here are exclusively based on the use of Eq. (16) with Ψ_i obtained from the EC3 buckling curves – this means that they may not reflect the actual physical behaviour of real frames.

2.4.2 Parametric study

In order to assess the validity of the "critical column concept", a parametric study was carried out for the simple plane structural systems (or "frames") shown in Figure 6: (a) a simple construction unbraced frame with a rigid beam and (b) an L-shaped frame with both members compressed. The aim of this study consists of comparing the strength estimates obtained by means of the critical column concept with "exact" ultimate (collapse) loads Λ_u, yielded by second-order plastic-zone finite element analyses, performed in the code ABAQUS [7]. The two flexible members in each frame (members 1 and 2) are HEB300 hot-rolled profiles made of S235 carbon steel and are acted by axial compressive forces P_1 and P_2. Only in-plane (major axis bending) behaviour was allowed and several geometry/loading combinations were considered, identified by the values of μ (length ratio), ρ (axial load ratio) and $\bar{\lambda}_f$ (frame slenderness), parameters defined as

$$\mu = \frac{L_1}{L_2} \qquad \rho = \frac{\overline{N}_1}{\overline{N}_2} \qquad \bar{\lambda}_f = \sqrt{\frac{\Lambda_y}{\Lambda_{cr}}} \tag{17}$$

For each pair of ρ–μ values, several L_2 and $L_1=\mu L_2$ values were considered, in order to obtain a sufficiently large set of $\bar{\lambda}_f$ values. The frames analysed ranged from rather "stocky" ($\bar{\lambda}_f = 0.2$) to quite "slender" ($\bar{\lambda}_f=3.0$) and over 280 cases were dealt with – all the selected pairs of ρ–μ values are given in Table 1[6].

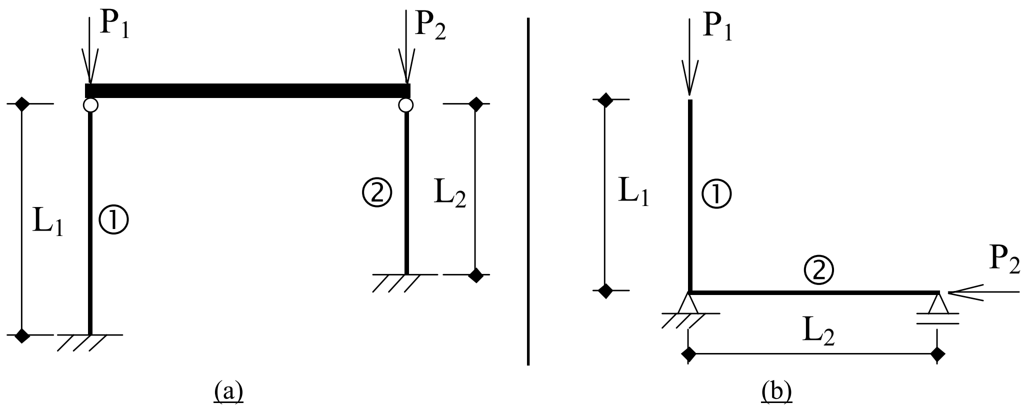


Figure 6. Structural systems employed in the parametric study: (a) simple unbraced frame with a rigid beam and (b) L-shaped frame

Table 1. Selected pairs of ρ–μ values: (a) simple unbraced frame and (b) L-shaped frame

ρ \ μ	**0.1**	**0.3**	**0.5**	**0.6**	**0.66**	**1**	**1.5**	**2**	**3**
0.5			(b)			(b)		(b)	
1	(a)	(a)	(a),(b)	(a)	(a)	(a),(b)		(b)	
1.5	(a)	(a)		(a)		(a)	(a)	(a)	(a)
2	(a)	(a)	(b)	(a)		(a),(b)	(a)	(a),(b)	(a)
3	(a)	(a)		(a)		(a)	(a)	(a)	(a)
10	(a)	(a)		(a)		(a)	(a)	(a)	(a)

6 Due to the particular geometry of the sway frame, only cases corresponding to ρ≥1 were dealt with.

The second-order plastic-zone analyses, performed in the finite element code ABAQUS [7], employed B31 beam elements and involved the specification of several cross-section integration points, indispensable to model the residual stresses distribution accurately [21]. All the frame members contained the initial bow imperfection and residual stress distribution shown in Figure 7 and an elastic-perfectly-plastic stress-strain law (*i.e.*, with no strain hardening) was adopted to model the steel material behaviour (E = 210 GPa, ν = 0.3 and f_y = 235 MPa). In order to avoid unnecessary modelling difficulties, the cross-section web-flange radii were neglected.

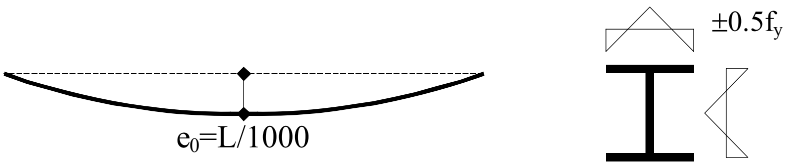


Figure 7. Frame member initial geometrical imperfection and residual stress distribution

The results obtained for the two frames are summarized in the diagrams presented in Figure 8. Each diagram shows all the results concerning each frame and provides the variation of the frame ultimate load reduction factor $\chi_f=\Lambda_u/\Lambda_y$ with $\bar{\lambda}_f$. Also depicted in each diagram are (i) the EC3 buckling curve governing the major axis flexural buckling behaviour of an HEB300 profile (curve *b*) and (ii) the "ideal" column/frame curve. Before analysing the results, one should recall that, since all columns exhibit *identical* cross-sections (same geometry, steel grade and buckling curve), the critical column is the one acted by the higher axial force – *i.e.*, column 1 (column 2) if ρ≥1 (ρ≤1). Moreover, the frame reduction factor χ_f is always equal to its critical column counterpart χ_c, *i.e.*, one has

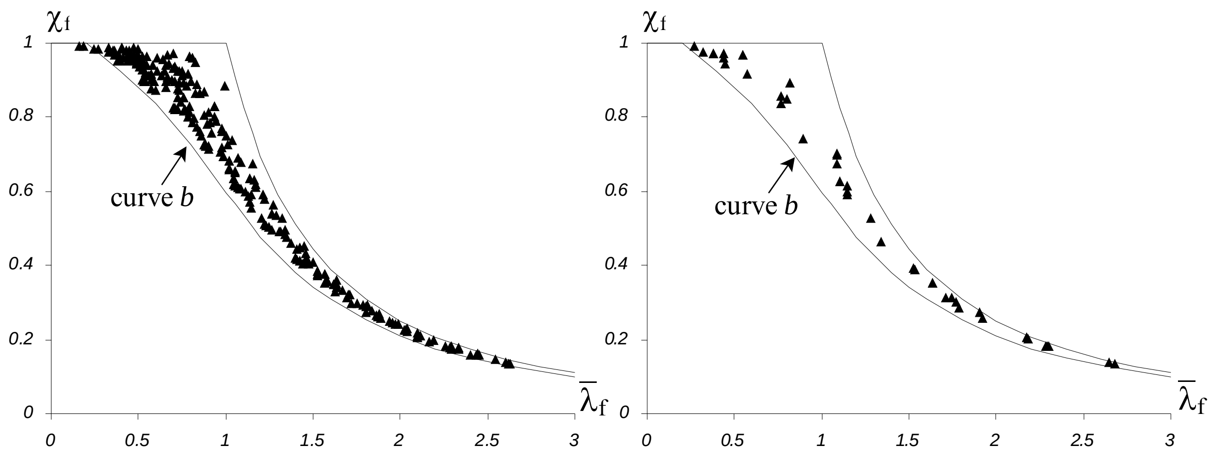
$$\chi_f = \frac{\Lambda_u}{\Lambda_y} = \frac{\Lambda_u}{\min(Af_y/\overline{N}_i)} = \max\left(\frac{\Lambda_u \overline{N}_i}{Af_y}\right) = \frac{N_{u,c}}{Af_y} = \chi_c \tag{18}$$

which stems from the equivalence of the "critical column" and "frame stability curve" concepts, whenever the same buckling curve is adopted for all columns – recall subsection 2.4.

The observation of the results displayed in Figure 8 prompts the following remarks:

(i) Virtually all results lie "not too much" above the buckling curve *b*, which means that the use of this buckling curve leads to *conservative* and *fairly accurate* strength estimates. All the (very few) unsafe predictions occur for $\bar{\lambda}_f$ <0.2 and stem mostly from neglecting strain-hardening. On the other hand, some excessively conservative estimates take place for either $\bar{\lambda}_f \approx 1$ or "odd" frame geometries (*e.g.*, μ=0.1) – although these particular estimates cannot be sorted out from Figure 8, all the separate results are available in reference [21].

(ii) In view of the above results, it is fair to say that, at least for the particular cases investigated, the concepts of "critical column" and "frame stability curve" have been validated. This means that, if one performs only the safety check of the *critical column*, by means of the buckling curve that governs its behaviour (curve *b* in this case), one is led to accurate and safe *frame* strength estimates and, therefore, no other column needs to be checked. It is worth noting that, as anticipated in subsection 2.4, this statement also holds true if the critical column is the shorter one – all the cases in which one has either (i) ρ>1 and μ<1 or (ii) ρ<1 and μ>1.



(a) (b)

Figure 8. Critical column results: (a) simple unbraced frame and (b) L-shaped frame

Finally, one last word concerning the reduction factor of the non-critical column χ_{nc}, which has not been plotted in Figure 8 and can be calculated on the basis of χ_c, by using the relations

$$\frac{\chi_{nc}}{\chi_c} = \frac{N_{u,nc}}{A\,f_y}\frac{A\,f_y}{N_{u,c}} = \frac{\overline{N}_{nc}}{\overline{N}_c} = \min(\rho\,;1/\rho) \tag{19}$$

Therefore, one has always $\chi_{nc}<\chi_c$, which is a direct consequence of the fact that the non-critical column is acted by a lower axial force at collapse.

2.4.3 Particular case: braced frames

Let us now address the particular case of columns integrated in braced (non-sway) frames, for which it is routine practice to assume that their buckling lengths can be conservatively taken as equal to the distance between the corresponding lateral supports ("system length") − for instance, such a simplifying assumption is stated in clause 5.5.1.5(1) of the EC3-ENV [3][7]. However, one should be aware that this approach is potentially dangerous when the critical member N_{cr} value is overestimated, because it leads to lower (and, therefore, *unsafe*) $\overline{\lambda}_c$ values − recall the inverted L-frame depicted in Figure 1(a), for which the K-factor can be considerably higher than 1.

For illustrative purposes, one analyses next the in-plane behaviour (major axis bending) of the inverted L-frame with hinged supports depicted in Figure 9. Once again, the frame members are HEB300 profiles made of S235 steel. In order to have medium $\overline{\lambda}$ values, L = 10 m was adopted. By assuming K = 1 in both members, one is led to $\overline{\lambda}_1 = 0.819$, $\overline{\lambda}_2 = 1.639$, $\chi_1 = 0.7125$ and $\chi_2 = 0.2959$ and one would, therefore, believe that (i) the frame is able to withstand *any* loading if $N_{Ed} \leq N_{b,Rd}$ holds for both members and (ii) the *optimum* loading is characterised by

$$\overline{N}_2 = 1 \quad \overline{N}_1 = \frac{\chi_1}{\chi_2} = 2.408 \quad \rho = \frac{\overline{N}_1}{\overline{N}_2} = 2.408 \quad \Lambda_{Rd} = \frac{N_{b,Rd,i}}{\overline{N}_i} = 0.2959\frac{N_{pl}}{\gamma_{M1}} \tag{20}$$

However, the performance of a linear stability analysis of the whole frame yields, for this particular loading, $K_1 = 1.133$, $K_2 = 0.879$, $\overline{\lambda}_1 = 0.929$, $\overline{\lambda}_2 = 1.441$, $\chi_1 = 0.6428$, $\chi_2 = 0.3649$ and

---

[7] Note, however, that it does not appear in EC3-prEN [4].

$$\Lambda_{Rd} = \min\left(\frac{N_{b,Rd,i}}{\overline{N}_i}\right) = 0.2669\frac{N_{pl}}{\gamma_{M1}} \qquad (21)$$

which indicates that the assuming K = 1 leads to an overestimation of the frame ultimate strength by about 11%. Note also that one has K_1=1.133 >1 for the critical member (member 1, since $\bar{\lambda}_f = \bar{\lambda}_1 < \bar{\lambda}_2$), which corresponds precisely to the potentially dangerous situation identified above. Moreover, higher errors are obtained if the length of member 1 is reduced – *e.g.*, 19% for L_1 = 5 m and 24% for L_1 = 2 m.

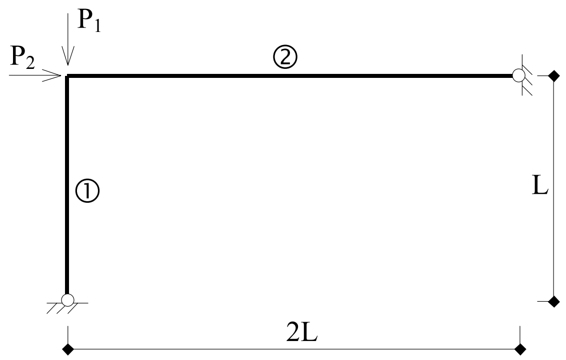


Figure 9. Braced frame illustrative example: inverted L-frame with hinged supports

The curves depicted in Figure 10 represent all the pairs of normalised ultimate load values $N_{u,1}/N_{pl}$–$N_{u,2}/N_{pl}$ obtained by (i) using curve *b* and K=1 in both members, (ii) using curve *b* and "exact" K values (yielded by frame linear stability analyses) and (iii) performing in ABAQUS second-order plastic zone ("exact") analyses that incorporate member imperfections (see subsection 2.4.2). Moreover, in order to ensure a meaningful comparison between these three approaches, γ_{M1}=1 was adopted in the first two.

After observing the results presented in Figure 10, it is possible to conclude that:

(i) Assuming K = 1 leads to the dashed horizontal and vertical lines and, as already mentioned, the ultimate load of each member becomes independent of the loading (ρ value).
(ii) The comparison between the "K = 1" and "exact K" curves shows that the former approach (ii_1) mostly leads to extremely conserv ative strength estimates (particularly if N_1 is small), but (ii_2) also yields unsafe results in the vicinity of ρ = 2.408.

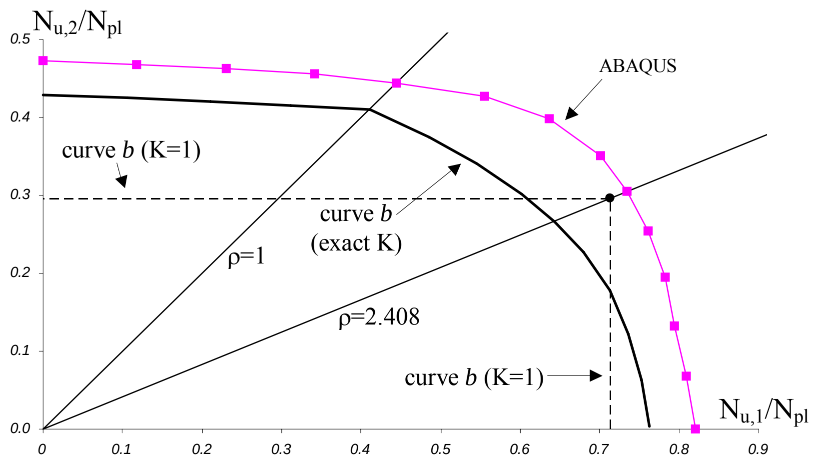


Figure 10. Inverted L-frame ultimate load values

(iii) Since the frame is statically indeterminate and the buckling curve always underestimates the actual frame collapse load (see Figure 8), the "exact" analyses ("ABAQUS" curve) are somewhat higher than the "exact K" strength estimates and considerably higher than the "K = 1" ones. This fact "masks" all the (theoretical) errors associated with the adoption of K = 1 − the authors investigated a large number of different frame geometries and loadings and found only a few slightly unsafe results, all of them corresponding to frames with odd geometries (*e.g.*, one member much longer that the other).

(iv) In view of the above items, it can be successfully argued that the choice of K = 1, although theoretically incorrect, does not lead to unsafe designs for situations of practical interest.

(v) Finally, it is still important to mention that the "kink" exhibited by the "exact K" curve at $\rho = 1$ is due to the fact that this point corresponds to a critical member switch: member 1 is critical for $\rho>1$ and member 2 for $\rho<1$ (obviously, both members are critical for $\rho = 1$).

3. SAFETY CHECKING OF BEAM-COLUMNS INTEGRATED IN FRAMES

3.1 The Method 1 and Method 2 Beam-Column Interaction Formulae

As far as Europe is concerned, the ongoing research on the design and safety checking of steel beam-columns has been taking place in the context of the activities of the Technical Committee 8 ("Stability") of the European Convention for Constructional Steelwork (ECCS). This research work led to the inclusion of two distinct sets of interaction formulae in EC3-prEN [**4**], which are identified by the designations "Method 1" and "Method 2"[8]. The Method 2 formulae, which were developed by an Austrian-German research team [**22-24**], aim mostly at a simple and user-friendly format and, thus, involve only a fairly small number of fully *empirical* parameters − they have no physical meaning and values are obtained through calibration procedures, *i.e.*, the comparison with numerically (mostly) and experimental results. On the other hand, the aim of the Method 1 formulae, developed by a French-Belgian research team [**25-27**], are two-fold: (i) to achieve a higher accuracy and also (ii) to assign a clear physical meaning to as many parameters as possible. It seems fair to say that there is a kind of "trade-off" between simplicity (Method 2) and accuracy/transparency (Method 1). In the particular case of the in-plane behaviour of compact (class 1 or 2) I-section beam-columns subjected to major axis bending, the Method 1 and Method 2 interaction formulae read

Method 1:
$$\frac{N_{Ed}}{\chi_y A\, f_y/\gamma_{M1}} + \frac{C_{my,1} M_{y,Ed}}{(1-\chi_y N_{Ed}/N_{cr,y}) C_{yy} W_{pl,y}\, f_y/\gamma_{M1}} \le 1$$

$$C_{yy} = 1 + (w_y - 1)\left(2 - \frac{1.6}{w_y} C_{my,1}{}^2 (\bar{\lambda}_y + \bar{\lambda}_y{}^2)\right) \frac{N_{Ed}}{A f_y/\gamma_{M1}} \ge \frac{W_{el,y}}{W_{pl,y}} \quad w_y = \frac{W_{pl,y}}{W_{el,y}} \le 1.5 \qquad (22)$$

Method 2:
$$\frac{N_{Ed}}{\chi_y A f_y/\gamma_{M1}} + \frac{C_{my,2} M_{y,Ed}}{W_{pl,y} f_y/\gamma_{M1}} \left(1 + \frac{N_{Ed}}{A f_y/\gamma_{M1}}\right) \min(\bar{\lambda}_y - 0.2;\, 0.8) \le 1 \qquad (23)$$

where $W_{el,y}$ and $W_{pl,y}$ are the cross-section elastic and plastic moduli. Besides satisfying one of the above interaction formulae, the resistance of the member *end sections* must be checked by using

[8] The reader should be warned that these sets of beam-column interaction formulae have been often designated in the literature as "Level 1 formulae" and "Level 2 formulae". Unfortunately, the former correspond to "Method 2" and the latter to "Method 1" – failing to take this "switch" into account will lead to considerable misunderstanding.

appropriate N-M plastic interaction formulae. However, when applying the interaction formulae defined by Eqs. (22) and (23) to frame members, the following two aspects deserve special attention:

(i) Although it was demonstrated that, in frames having only axially compressed members, the "critical column" may be easily and quickly identified, such feature no longer exists when the frame members are also subjected to bending moments (*i.e.*, are beam-columns). This fact raises the question of whether it is necessary to calculate the exact buckling lengths required to apply the beam-column interaction formulae, particularly in the case of members acted by small axial forces – recall that large L_{cr} and, therefore, low χ values will be obtained. This issue will be further addressed later in this paper.
(ii) The accuracy of the above interaction formulae is strongly dependent on the choice of an appropriate equivalent moment factor value C_{my}. However, it is very important to emphasise that the Method 1 and Method 2 formulae were developed on the basis of slightly different "equivalent moment" concepts. Indeed, for simply supported members, $C_{my,1}$ and $C_{my,2}$ concern *sinusoidal* and *uniform* equivalent moments, which means that one has, respectively[9],

$$C_{m,1} = (1 - N_{Ed} / N_{cr})\frac{M_{Ed}^{II,max}}{M_{Ed}^{I,max}} \quad C_{m,2} = \cos\left(\frac{\pi}{2}\sqrt{N_{Ed} / N_{cr}}\right)\frac{M_{Ed}^{II,max}}{M_{Ed}^{I,max}} \tag{24}$$

where $M_{Ed}^{I,max}$ and $M_{Ed}^{II,max}$ are the maximum first and second-order moments acting on the beam-column, for a given axial load N_{Ed}. Both the Method 1 and Method 2 formulae include a more or less extensive set of C_m expressions or values, all of which have been derived in the context of isolated members (mostly simply supported) and, therefore, cannot be readily applied to members integrated in frames. In order to circumvent this difficulty and avoid the need to evaluate accurate C_m values, an alternative approach is suggested here [**28**]: to incorporate each of the theoretical C_m expressions defined by Eq. (24) directly into the associated interaction formulae (22)-(23), thus yielding

<u>Method 1</u>:
$$\frac{N_{Ed}}{\chi_y A f_y / \gamma_{M1}} + \frac{(1 - N_{Ed} / N_{cr,y}) M_{y,Ed}^{II}}{(1 - \chi_y N_{Ed} / N_{cr,y}) C_{yy} W_{pl,y} f_y / \gamma_{M1}} \leq 1$$

<u>Method 2</u>:
$$\frac{N_{Ed}}{\chi_y A f_y/\gamma_{M1}} + \frac{\cos(\frac{\pi}{2}\sqrt{N_{Ed}/N_{cr,y}}) M_{y,Ed}^{II}}{W_{pl} f_y/\gamma_{M1}}\left(1 + \frac{N_{Ed}}{\chi_y A\, f_y/\gamma_{M1}}\right)\min(\bar{\lambda}_y - 0.2; 0,8) \leq 1 \tag{25}$$

In these formulae, it is implicitly assumed that $M_{y,Ed}^{II}$ is the *maximum* bending moment acting on the beam-column (*i.e.*, the superscript "max" is omitted), which is obtained from a frame second-order global analysis that *does not incorporate* member imperfections – they are already accounted for by the interaction formulae[10]. An additional benefit of this approach is related to the fact that either of the new formulae tend to the commonly used cross-section plastic moment check ($M_{y,Ed}\gamma_{M1}/W_{pl,y}f_y \leq 1$) as N_{Ed} approaches zero – this is in contrast with the original formulae, defined by Eqs. (22) and (23), which become

$$\frac{C_{my} M_{y,Ed}}{W_{pl,y} f_y / \gamma_{M1}} \leq 1 \tag{26}$$

leading to the need to perform an additional end cross-section plastic checks when $C_{my} \leq 1$.

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[9] An in-depth discussion about this matter can be found in a recent paper by the authors [32].

[10] If the member second-order axial forces differ substantially from their first-order counterparts (not the usual case), one should also replace N_{Ed} by N_{Ed}^{II}.

3.2 Parametric study

In order to assess the performance of the proposed beam-column interaction formulae and also to investigate whether it is necessary to calculate exact buckling lengths, a second parametric study was carried out concerning the two simple frames analysed earlier and depicted in Figure 6. However, the frame loading now includes a lateral load Q that induces exclusively first-order bending moments in the members – see Figure 11. It is worth mentioning that the rather peculiar L-shaped frame was selected because of one specific and rather interesting feature: no first-order moments appear in member 1 (only second-order ones).

In this study, the strength estimates obtained by means of the proposed interaction formulae (25) are compared with "exact" results, obtained once more from second-order plastic-zone analyses performed in ABAQUS, which include the initial imperfections shown in Figure 7 in all members. Since the proposed formulae are based on second-order moments, it was also necessary to perform elastic second-order analyses of the two frames, a task that was undertaken resorting to a standard matrix structural analysis method using stability functions (*e.g.*, [29,30]). One should still mention that both frames are classified, according to EC3-prEN, as "unbraced", which means that appropriate frame geometrical imperfections (or, alternatively, equivalent lateral forces) should be included in their global analyses. However, for the sake of simplicity and without any loss of generality (only the loading would change, due to the additional equivalent lateral forces), the influence of this type of geometrical imperfections was disregarded in this study.

Due to space limitations, only a selected (but representative) set of the results is presented in Figures 12 and 13, respectively for the simple unbraced and L-shaped frames – all the remaining results can be found in [31]. This set of results corresponds to the ρ–μ combinations given in Table 2 (recall Eqs. (17)) and it is worth noting that they are all Massociated with situations such that (i) member 1 is acted by the higher axial force (ρ>1) and (ii) the maximum first-order bending moment occurs in member 2. As before, for each pair of ρ–μ values, several L_2 and $L_1=\mu L_2$ values were considered, thus making it possible to cover a sufficiently wide range of $\bar{\lambda}_f$ values. Moreover, in order to keep the number of parameters involved low, it was decided (i) to take γ_{M1} = 1.0 in the interaction formulae and (ii) to adopt "exact" χ values in member 1 (*i.e.*, the one subjected to the higher axial force), extracted from the plastic-zone results presented in Figure 8. Concerning the last issue, it is obvious that the use of the column buckling curves would lead to slightly lower χ values, thus making the interaction formulae predictions somewhat safer.

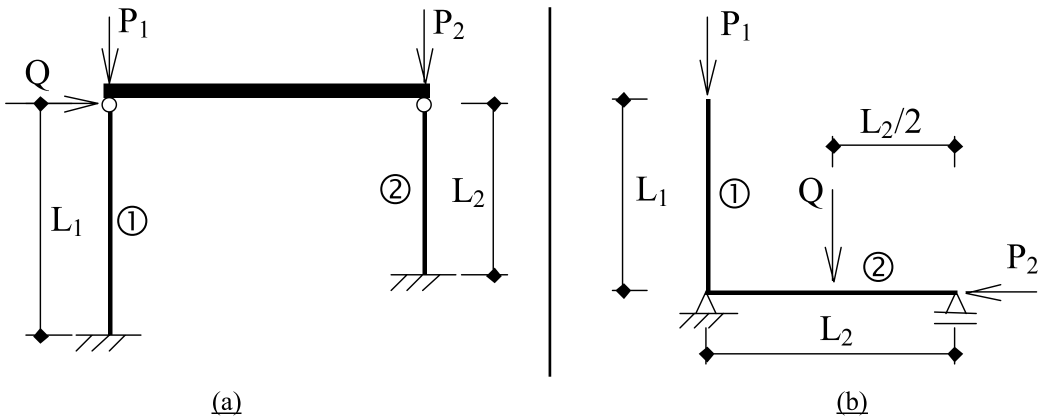

Figure 11. Structural systems employed in the beam-column parametric study:
(a) simple unbraced frame with a rigid beam and (b) L-shaped frame.

Table 2. Selected pairs of ρ–μ values: (a) simple unbraced frame and (b) L-shaped frame

ρ \ μ	**0.5**	**1**	**2**	**3**
1.5		(a)	(a)	(a)
2	(b)	(b)	(b)	
3		(a)	(a)	(a)

Each diagram in Figures 12 and 13 corresponds to a particular pair of ρ–μ values and provides, for several $\bar{\lambda}_f$ values, the pair of ultimate Q–P_1 load values. The observation of the results leads to the following conclusions:

(i) The application of the proposed formulae (Eqs. (25)), which employs (i_1) exact buckling lengths (obtained from linear stability analyses) and (i_2) elastic second-order moments (obtained from genuine second-order global analyses), consistently leads to rather accurate frame ultimate strength estimates[11]. As expected, the Method 1 formula provides the more accurate load-carrying capacity predictions[12], even if the differences between the two Methods are always rather minute – a similar assessment was made when both formulae were applied to isolated members, employing exact C_{my} values [32].

(ii) For low axial forces, the application of either formula leads to rather conservative strength estimates. This is due to the fact that the second-order effects are almost negligible and, therefore, the structures are able to withstand a loading level very close to the one associated with the occurrence of a first-order rigid-plastic (plastic hinge) collapse mechanism [21]. Naturally, this effect is more relevant in frames exhibiting low $\bar{\lambda}_f$ values[13].

(iii) Finally, note that the majority of the (few) unsafe estimates concern the L-shaped frame and correspond to situations associated with high $\bar{\lambda}_f$ and axial force values, *i.e.*, situations in which moderate-to-large displacements are bound to occur – obviously, such situations are outside the range of validity of the elastic second-order global analyses performed (they were based on the use of standard stability functions).

[11] Note that this statement also applies to the L-shaped frame, where no first-order bending moments act in member 1. This would certainly lead to some difficulties, concerning the application of the original formulae – recall that Eqs. (22) and (23) involve first-order bending moments.

[12] Recall that Method 2 was developed with the explicit objective of achieving a high degree of simplicity and user-friendliness, even at the cost of some accuracy and transparency.

[13] Concerning this aspect, it is worth mentioning that the EC3-prEN states explicitly that the buckling effects may be ignored in columns for which $N_{Ed}/N_{cr} \leq 0.04$ – only the cross-section resistance needs to be checked.

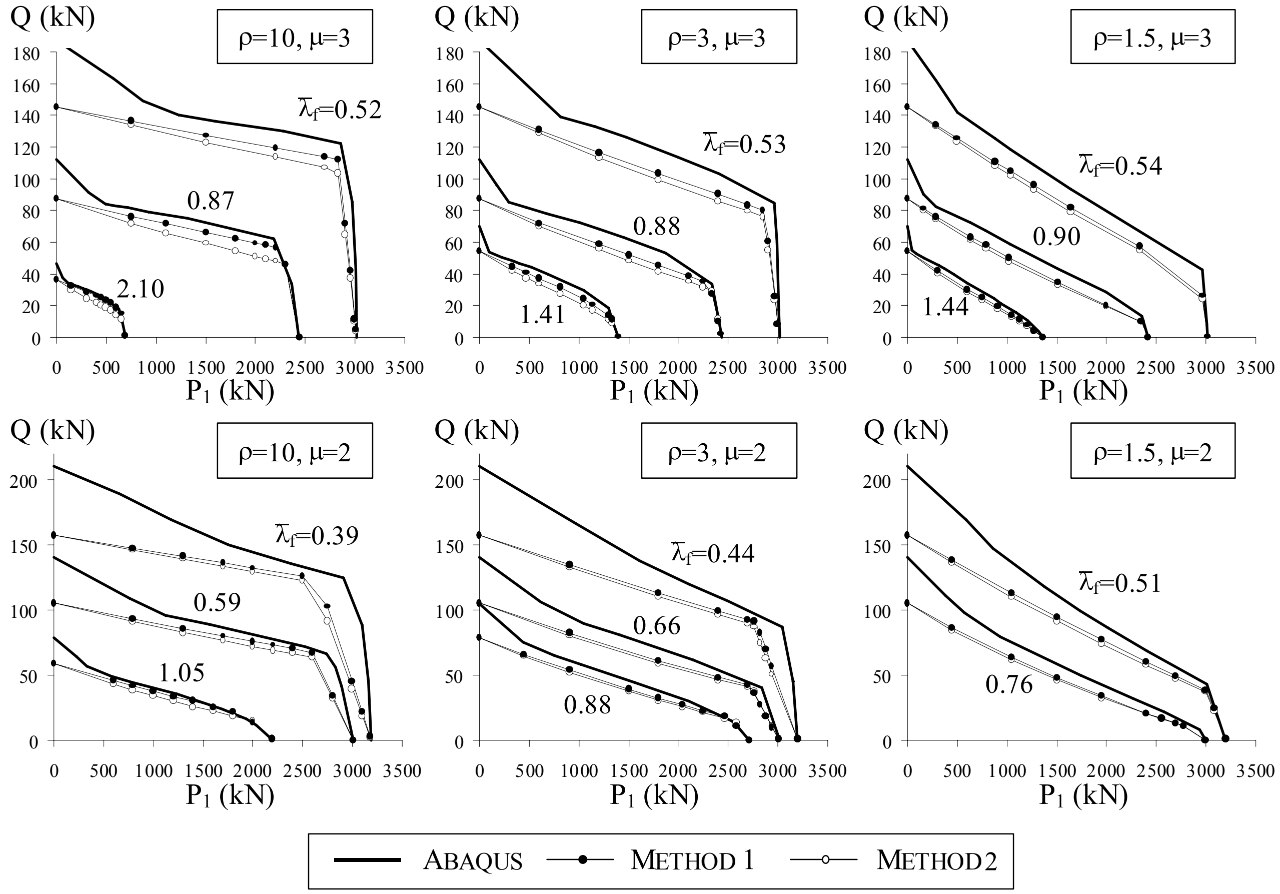

Figure 12. Beam-column parametric study: simple unbraced frame results

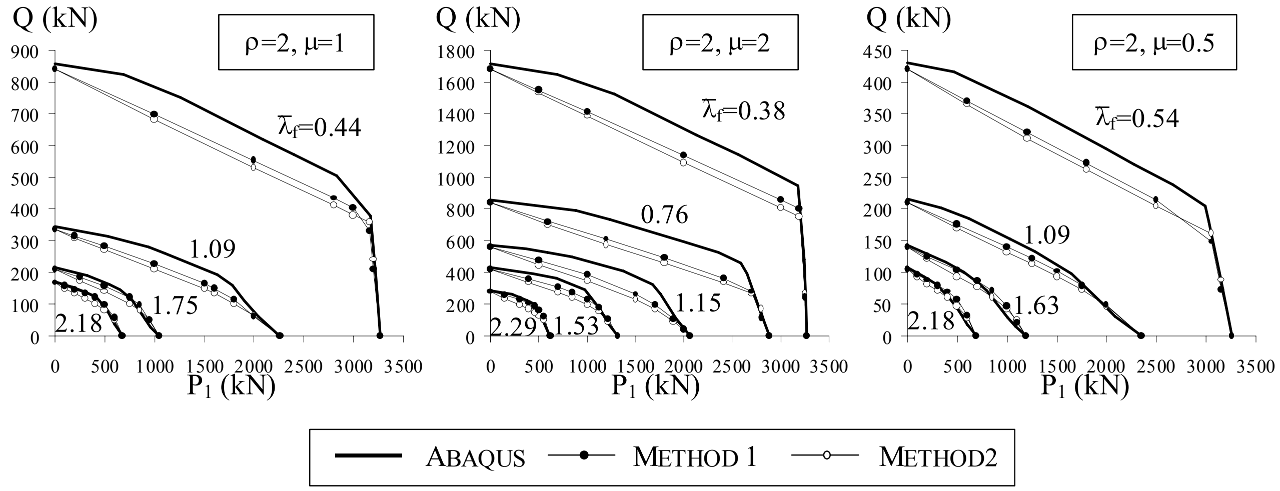

Figure 13. Beam-column parametric study: L-shaped frame results

It is still worth mentioning that if one chooses to employ Eqs. (22) and (23), adopting tabulated C_m values, rather than the proposed formulae given in Eq. (25), extreme caution is required since, as mentioned earlier, the overwhelming majority of the available C_m values and expressions were determined for *isolated* and *simply supported* beam-columns [32] – the misuse of these values and expressions may lead to considerably unsafe strength estimates [31].

Finally, a few words concerning the choice of the buckling lengths to be incorporated in the beam-column interaction formulae. The results displayed in Figures 12 and 13 clearly show that accurate frame ultimate load values are consistently obtained if one adopts exact L_{cr} values, even when these values are very large and "non intuitive", *e.g.*, when $\rho>>1$ – from Eq. (7), one obtains $L_{cr,2}=\sqrt{\rho}L_{cr,1}$, which means that, for instance, one has $\rho=10$ and $L_{cr,2}=\sqrt{10}L_{cr,1}$ if $\overline{N}_1=10\overline{N}_2$. Although the authors have previously shown that underestimating such large L_{cr} values may lead to extremely unsafe strength estimates [21,31] [14], it should be made absolutely clear that the underestimation of L_{cr} corresponds, in fact, to an overestimation of N_{cr}, which often leads to serious safety problems. Therefore, the use of exact, albeit very large, buckling lengths is strongly recommended.

4. CONCLUSION

In this work, one discussed some fundamental concepts and presented a few illustrative results concerning the design and safety checking of columns and beam-columns integrated in plane steel frames – particular attention being paid to the provisions prescribed by the upcoming (European Norm) version of Eurocode 3. Initially, the paper addressed the ambiguities and surprising results that a designer may be faced with when he wishes to calculate the buckling lengths of compressed members belonging to frame structures. Besides clarifying a number of peculiar issues and showing that the "exact" buckling length values must be obtained from frame linear stability analyses, it was also demonstrated that a *theoretically sound* member safety checking procedure must necessarily incorporate those "exact" buckling lengths – moreover, the use of "intuitive" values (prompted by an "isolated member reasoning") may lead to considerably unsafe frame strength estimates.

Next, one introduced the concept of frame "critical column", *i.e.*, the column that governs the frame strength and safety checking procedure, which (i) provides valuable insight into the frame overall behaviour (for instance, by making it easy to spot the frame "weak" members) and (ii) may be used to develop a frame optimisation procedure. After addressing the identification of the "critical column" of a given frame, it was shown that its normalised slenderness always supplies the value of the frame normalised slenderness, provided that all the frame columns are governed by the same buckling curve.

Finally, attention was devoted to the design and safety checking of beam-columns integrated in frames using the novel (Method 1 and Method 2) interaction formulae included in the upcoming version of Eurocode 3. It was shown that, at least for the simple frames dealt with in this paper, the inclusion of "exact" buckling lengths and equivalent moment factors (C_m) consistently leads to frame ultimate load-carrying capacity predictions that are both safe and accurate. However, since (i) the accuracy of the strength estimates provided by the formulae is strongly dependent on the choice of appropriate C_m values and (ii) it is rather difficult to obtain such values for frame members, an alternative approach to use the above formulae was proposed: to incorporate genuine second-order elastic bending moments, yielded by the analysis of "ideal" (initially perfect) frames.

In order to assess the validity and/or efficiency of the various concepts and procedures addressed in this work, several illustrative examples were presented and discussed throughout the paper. They dealt with simple two-bar frames and the numerical results obtained were compared with "exact" values, yielded by second-order plastic zone finite element analyses, performed by means of the code ABAQUS

[14] In particular, the following "intuitive" K-factors were considered: (i) K=2 for the simple unbraced frame member acted by the smaller axial load and (i) K=1 for the L-shaped frame horizontal member. In both cases, the interaction formulae yielded several rather unsafe strength estimates.

and including standard initial geometric imperfections and residual stresses – the proposed methodologies were shown to consistently yield safe and accurate strength predictions.

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